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UNIVERSITE MUSTAPHA STAMBOULI DE MASCARA
FACULTÉ DES SCIENCES ET TECHNOLOGIE



Course Handout

Heat Transfer

**Presented by: KHELIL
FOUDIL**

This course is intended for undergraduate students in Mechanical Engineering under the LMD system."

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FOREWORD

This course is aimed at end-of-cycle LMD students in the mechanical engineering discipline.

The energy consumed worldwide primarily comes from two sources: the combustion of fossil reserves (such as oil, coal, and natural gas) and renewable resources (such as solar, wind, hydroelectric, and biomass energy). Regardless of the technologies used, the mastery of nuclear energy, solar thermal energy, deep geothermal energy, or heat pumps relies heavily on the principles of thermal transfers.

Thermal transfers also play a crucial role in the efficiency of propulsion systems, energy production systems, and most industrial or everyday systems, such as electronic devices. Effective thermal management is essential to optimize the performance and durability of these systems. Therefore, thermal transfers constitute a key science for the energy of the future.

This course is based on the professional experience of the author, a teacher at the University of Mustapha Stambouli of Mascara. It is organized into three chapters:

1. First Part: Approach to Thermal Transfers by Conduction

- Intended for third-year undergraduate students, this part offers a comprehensive introduction to the discipline with minimal formalism. The different modes of transfer, including conduction and thermal radiation, are gradually introduced with a focus on the physical approach to phenomena. The goal is to make the concepts accessible without excessive complexity.

2. Second and Third Chapters: One-Dimensional Applications

- These chapters are dedicated to practical applications of thermal transfers, generally in one dimension, to simplify the complex mathematical and numerical aspects related to sophisticated geometries. This simplification allows students to focus on fundamental principles without being overwhelmed by complicated calculations.

3. Immediate Application Exercises - Throughout the chapters, practical solved exercises are included to illustrate theoretical concepts and facilitate understanding. These exercises are designed to reinforce learning by providing concrete and immediately applicable examples.

This course aims to provide students with a solid foundation in thermal transfers, essential for their education and future careers in the field of energy and thermal sciences.

I Generalities

1.1 Introduction to Heat Transfer

Heat transfer is one of the most common modes of energy exchange. It naturally occurs between two systems as soon as there is a temperature difference between them, regardless of the medium, even if it is a vacuum, that separates them. Thermal transfers play an essential role in various technological applications and become crucial when they are the basis of the techniques used (exchangers, thermal engines, insulation, use of solar energy...).

1.1.1 Examples of Thermal Machines

With a heat source and a cold source, it is always possible to recover energy. Examples abound. • Steam machines consume coal, which produces heat when burned. They all have a water reservoir, which constitutes the cold source. The water is vaporized by the heat released from the coal combustion and is put under pressure to move pistons, which means to provide energy. • Nuclear power plants are, in fact, thermal machines: their heat source is the nuclear reactor. They are placed next to a lake or river, which are slightly heated by the plant, to produce electricity.

1.1.2 Thermodynamics and Heat Transfer

At the foundation of the study of heat transfer are the concepts of heat quantity and temperature difference. These are defined by thermodynamics by the following principles: First

First principle

- Equivalence of heat and work as particular forms of energy. Second Principle

Second principle

- Measurement of the thermal imbalance of two systems by their temperature difference, with the value of this difference characterizing the direction and intensity of the transferred heat energy. Therefore, a heat transfer corresponds to a variation in internal energy. Thermodynamics, however, only deals with equilibrium states, neglecting the exchange mechanisms that lead to them. The study of these mechanisms has thus developed in parallel and, due to its importance, with sufficient breadth to constitute an independent discipline: heat transfer.

There are three modes of heat transfer that can occur independently or simultaneously:

- Conduction.
- Radiation.
- Convection.

1.2 Quantities and Units

1.2.1 Temperature

In an object, the molecules, the atoms that constitute it move extremely rapidly; this disorderly motion of particles is called thermal agitation. Temperature measures the degree of

agitation of the particles: the more agitated the molecules of an object are, the hotter it is. Temperature is measured in degrees Celsius, Kelvin, or Fahrenheit.

Celsius - Fahrenheit

$$T_C = \frac{5}{9}(T_F - 32) \quad (1-1)$$

The absolute speed of a material point M in R is written:

$$\vec{v}_{/R}(M) = \vec{v}_{/R}(M \in R_1) + \vec{v}_{/R_1}(M) \quad (2-3)$$

CELCIUS - KELVIN

$$T_K = T_C + 273,15 \quad (1-2)$$

Then, at each point M of a body (solid, liquid, or gas), a temperature is defined, a scalar function of the coordinates of the point and time. $\theta(M, t)$. The set of points having the same temperature at each instant is called an isothermal surface.

1.2.2 Quantity of Heat

Heat represents the kinetic energy of molecules and atoms of matter. The measurement of heat quantities, or calorimetry, is based on measuring heat transfers. The unit of heat quantity is the calorie or joule (1cal = 4.18 joules); by definition, 1 calorie is the quantity of heat required to raise one gram of water by one degree Celsius.

1.2.3 Heat Flux or Heat Power

This is the amount of heat exchanged through a surface per unit of time (unit: W).

$$\phi = \frac{\partial Q}{\partial t} \quad (1-3)$$

1.3 Heat Flux Density

This is the amount of heat exchanged per unit area and per unit of time (unit: W/m²).

$$\phi = \frac{Q}{S} \quad (1-4)$$

S: Exchange surface

1.4 Different Regimes

The heat transmission regime may or may not vary over time. The regime is said to be steady-state (or permanent) if the temperature θ existing at each point of the body remains the same over time.

Steady-state regime: The temperature does not vary over time. (Established regime):: $\frac{\partial \theta}{\partial t} = 0$

This is a partial derivative because the temperature varies from one point to another while remaining constant at each point.

A transient regime (or non-steady-state) is characterized by a time evolution of the temperature at each point of the body; in practice, this occurs during heating or cooling of bodies.

Transient regime: The temperature varies with time: $\frac{\partial \theta}{\partial t} \neq 0$

Note: In a variable regime, isothermal surfaces are mobile and deformable; in a steady-state regime, they are invariant.

1.5 Different Heat Transfer Modes

1.5.1 Conduction

Conduction is the phenomenon where heat flows within a medium from a region of high temperature to a region of low temperature. Energy propagates through direct contact of particles without appreciable displacement. It is the only mechanism through which heat can flow within a solid body.

1.5.2 Radiation

Radiation is the mechanism by which heat transfers from a body at high temperature to one at lower temperature, even across a vacuum. Radiation involves wave phenomena that transport energy. Radiated heat travels at the speed of light (3.108 m/s in vacuum). Any body above 0 K emits radiation in the form of electromagnetic waves. The human body regulates heat loss to maintain its temperature at 37°C.

- A resting subject radiates about 75 W
- A subject in normal activity while seated radiates about 100 W
- A person engaged in physical effort radiates about 150 W

1.5.3 Convection (Solid-Fluid)

Convection corresponds to the movement of a fluid due to temperature variation. Forced convection occurs when fluid movement is induced by external action (pump, fan, etc.), while natural convection arises simply from density differences between hot and cold parts of the fluid.

1.6 Combinations of Different Transfer Modes

Above, we have considered the three basic modes of heat exchange separately. In reality, these modes are often closely intertwined: conduction and radiation in the case of non-opaque solids (such as glass, plastics), or conduction, convection, and radiation in fluids.

Furthermore, any heat exchange resulting in temperature changes of the involved bodies often involves phase changes (vaporization, condensation, melting, freezing), which act as additional sources or sinks of heat.

Example:

When heating water in a container over a flame, the energy released during water evaporation involves:

- Convection and radiation in the transfer between the hot gases from the flame and the external wall of the container.
- Conduction through the container wall and the adjacent fluid layers.
- Convection and some conduction within the water mass.
- Upon sufficient heating, boiling and subsequent vaporization become essential elements of heat exchange.

As shown in the example above, most technical problems involve a combination of different heat transfer modes. Typically, one mode predominates, leading to the neglect of others, or multiple modes are equally important but can be decoupled and treated separately. When these approaches are not feasible, numerical methods are necessary for analysis.

2. Heat Transmission by Conduction

Consider a solid body that is homogeneous and isotropic (its physical properties are the same in all directions of space), through which a unidirectional heat current passes.

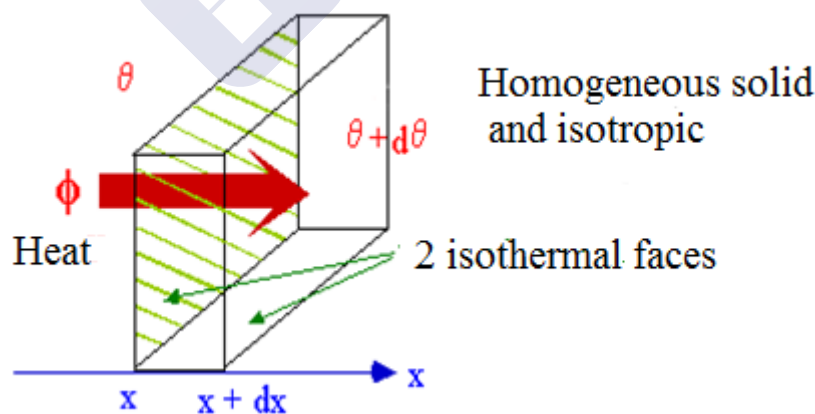


Figure 2.1. Conduction in an Elementary Layer of a Flat Wall

Imagine a small plane layer perpendicular to the x -direction of heat propagation, with thickness dx and area S within this medium. The two faces of this layer are isothermal surfaces. The first face is at temperature θ , and the second face is at temperature $\theta + d\theta$ (where $d\theta < 0$).

The temperature gradient, which is the temperature variation per unit length in the direction of heat propagation, is defined as:

The thermal flux through the plane layer of area S , proportional to the temperature gradient, is given by:

$$\Phi = -\lambda \cdot S \cdot \frac{d\theta}{dx} \quad (2-1)$$

This is Fourier's law of heat conduction.

The proportionality coefficient λ is the thermal conductivity of the material, which depends on the material and its temperature. λ is expressed in units of $\text{W} \cdot \text{m}^{-1} \cdot \text{K}^{-1}$ in the International System (SI) or $\text{kcal} \cdot \text{h}^{-1} \cdot \text{m}^{-1} \cdot \text{K}^{-1}$. Higher thermal conductivity indicates better heat conduction, whereas materials with lower thermal conductivity are used as insulators.

Among solids, metals are generally much more conductive than non-metallic compounds, with the exception of graphite (used in some heat exchangers). Stainless steel, for instance, has lower conductivity compared to most other metals and alloys.

Here is **Table 2.2** showing the thermal conductivity of various materials in units of $\text{W} \cdot \text{m}^{-1} \cdot ^\circ\text{C}^{-1}$:

Metals and Alloys (at room temperature)			
99.9% aluminum	228	Zinc	111
99% aluminum	203	Mild Steel (1% C)	46
99.9% copper	386	Stainless Steel (Cr 18% - Ni 8%)	16
Tin	61	Alloy (Al 92% - Mg 8%)	104
Pure Iron	85	Brass (Cu 70% - Zn 30%)	99
Pure Nickel	61	Titanium	21
Pure Lead	35		
NON-METALLIC SOLIDS (at room temperature)			
Asbestos (sheets)	0,162	Cork	0,046
Solid Concrete	1,7	Phenolic Plastics	0,046
Solid Fired Clay Bricks	1,16	Polyester Plastics	0,209
Fibre Cement Sheet	0,74	Polyvinyl Plastics	0,162
Ordinary Glass	0,70	Porcelain	0,928
Pyrex Glass	1,16	Glass Wool	0,046
Electrographite	116		
LIQUIDS		LIQUIDS AND GASES (at 0°C and normal pressure)	
Water at 20°C	0,59	Air	0,024
Water at 100°C	0,67	Nitrogen	0,024
Dowtherm A at 20°C	0,139	Acetylene	0,019
Benzene at 30°C	0,162	Hydrogen	0,174
Mercury at 20°C	8,47	Carbon dioxide	0,014
Sodium at 200°C	81,20	Oxygen	0,024

Regarding liquids:

- Mercury stands out prominently.
- Molten metals are good conductors, explaining, for example, the use of sodium salts as coolant fluids in nuclear reactors.

In general, the order of thermal conductivity is: λ of gases $<$ λ of liquids $<$ λ of solids. However, thermal conductivity varies with temperature:

- For solids, a linear approximation is often used: $\lambda = \lambda_0(1 + a\theta)$, where λ_0 is the thermal conductivity at 0°C , θ is the temperature, and a is a material-specific temperature coefficient.
 - $a > 0$ for many insulating materials.
 - $a < 0$ for most metals and alloys (except aluminium and brass).
- For liquids, thermal conductivity typically decreases with increasing temperature (except for water and glycerol).
- For gases, thermal conductivity generally increases with temperature.

2.1. Conduction through a homogeneous flat wall

Let a homogeneous flat wall, with area S and thicknesses, made of a material with an average thermal conductivity λ . One of the faces is at temperature θ_1 and the other at temperature θ_2

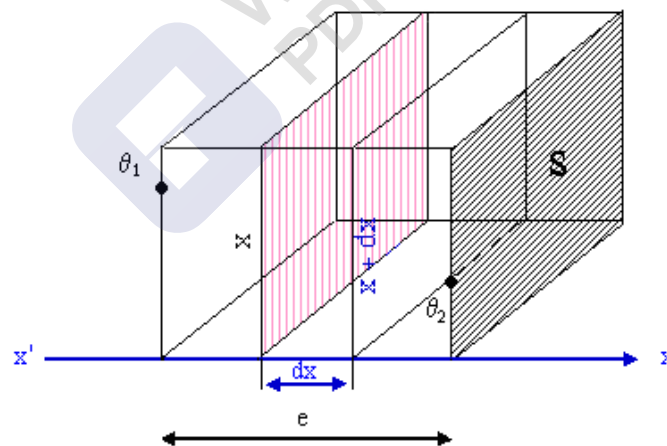


Figure 2.2 Conduction in a flat wall

2.1.1 Expression of the Thermal Flux of Conduction in a Planar Wall

If $\theta_1 > \theta_2$, a thermal flux flows by conduction through the wall from face 1 to face 2. The heat flow lines are straight and perpendicular to the isothermal faces 1 and 2. The lateral faces of the wall limit a flow tube, and the law of heat conservation allows us to write:

- Thermal flux Φ_1 = entering through face 1.

- Heat flux Φ = crossing any internal section parallel to the faces.
- Heat flux Φ_2 = exiting through face 2.

The thermal flux crossing by conduction through a thin layer of thickness dx located at a distance x from face 1, with its faces respectively at temperatures θ and $\theta + d\theta$ is given by FOURIER's law:

$$\Phi = -\lambda \cdot S \cdot \frac{d\theta}{dx} \quad (2-2)$$

$$\Phi \cdot dx = -\lambda \cdot S \cdot d\theta$$

After integration, we obtain:

$$\int_0^e dx = -\lambda \cdot S \int_{\theta_1}^{\theta_2} d\theta \quad \text{Either } \Phi \cdot e = -\lambda \cdot S \cdot (\theta_1 - \theta_2) \quad (2-3)$$

The thermal flux density is the flux per unit area:

$$\varphi = \frac{\lambda \cdot S \cdot (\theta_1 - \theta_2)}{e} \quad (2-4)$$

2.1.2 Expression of the Thermal Resistance of Conduction in a Planar Wall

As in electricity, resistance is the ratio of a potential difference (here, temperature) to an energy flow (here, the flux Φ), leading to the following expression for thermal resistance:

$$R = \frac{(\theta_1 - \theta_2)}{\varphi} = \frac{e}{\lambda \cdot S} \quad (2-6)$$

2.1.3 Application Example

- Calculate the flux passing through a glass pane with an area of 1 m^2 and a thickness of 3.5 mm . The temperature of the inner face of the glass is 10°C , and the temperature of the outer face is 5°C .
- Deduce the thermal resistance of the glass.
 - Thermal conductivity of glass: $\lambda_v = 0,7 \text{ W.m}^{-1}.\text{K}^{-1}$
- For the same wall temperatures, calculate the flux passing through 1 m^2 of a brick wall with a thickness of 26 cm . Deduce the thermal resistance.
- Thermal conductivity of bricks: $\lambda_b = 0,52 \text{ W.m}^{-1}.\text{K}^{-1}$.

Answer:

- Flux passing through 1 m^2 of glass:

$$\varphi = \frac{\lambda \cdot S \cdot (\theta_1 - \theta_2)}{e} = \frac{0,7 \cdot 1 \cdot (5 - 0)}{3.5 \cdot 10^{-3}} = 1000 \text{ W} \quad (2-7)$$

- Thermal resistance of 1 m^2 of glass:

$$R = \frac{e}{\lambda S} = \frac{3 \cdot 10^{-5}}{0,71} = 5 \cdot 10^{-3} \text{ } ^\circ\text{CW}^{-1} \quad R = \frac{(\theta_1 - \theta_2)}{\phi} = \frac{(10 - 5)}{1000} = 5 \cdot 10^{-3} \text{ } ^\circ\text{CW}^{-1}$$

- Flux passing through 1 m² of brick wall:

$$\phi_{brique} = \frac{\lambda S \cdot (\theta_1 - \theta_2)}{e} = \frac{0,52 \cdot 1 \cdot (10 - 5)}{0,26} = 10 \text{ W}$$

- Thermal resistance of 1 m² of brick wall:

$$R_{brique} = \frac{(\theta_1 - \theta_2)}{\phi} = \frac{(10 - 5)}{10} = 0,5 \text{ } ^\circ\text{CW}^{-1}$$

Analysis of Results

For the same area and temperature difference, the flux lost through the glass is 100 times higher than that lost through the brick wall, due to the lower conductivity and much greater thickness of the brick wall.

2.2 Conduction through Multiple Homogeneous Planar Walls in Series

Consider several walls bounded by parallel planes, made of materials with different conductivities but in perfect contact. Let $(\lambda_1, \lambda_2, \lambda_3)$ be the average thermal conductivities of each wall with thicknesses (e_1, e_2, e_3) respectively. As previously assumed, there are no lateral heat losses. Each wall is therefore traversed by the same thermal flux (Φ) .

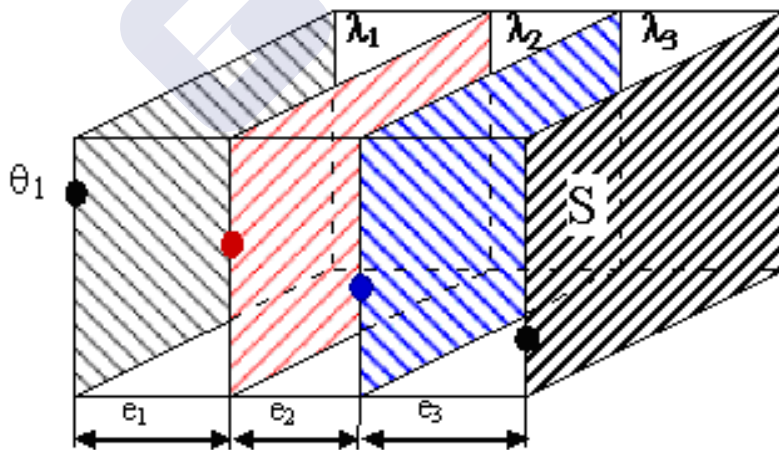


Figure 2.3 Conduction through Multiple Planar Walls in Series

2.2.1 Expression of the Thermal Flux of Conduction through Walls in Series

Based on the previous section, we can write the flux crossing each wall and deduce the temperature differences between the faces of each wall:

For wall 1:

$$\Phi = \lambda_1 \frac{S}{e_1} \cdot (\theta_1 - \theta_2) \Rightarrow (\theta_1 - \theta_2) = \frac{\Phi}{S} \cdot \frac{e_1}{\lambda_1} \quad (2-8)$$

For wall 2 :

$$\Phi = \lambda_2 \frac{S}{e_2} \cdot (\theta_2 - \theta_3) \Rightarrow (\theta_2 - \theta_3) = \frac{\Phi}{S} \cdot \frac{e_2}{\lambda_2} \quad (2-9)$$

For wall 3:

$$\Phi = \lambda_3 \frac{S}{e_3} \cdot (\theta_3 - \theta_4) \Rightarrow (\theta_3 - \theta_4) = \frac{\Phi}{S} \cdot \frac{e_3}{\lambda_3} \quad (2-10)$$

Since the same flux Φ passes through each wall in series, we can write:

$$\theta_1 - \theta_4 = \frac{\phi}{S} \cdot \left(\frac{e_1}{\lambda_1} + \frac{e_2}{\lambda_2} + \frac{e_3}{\lambda_3} \right) \quad (2-11)$$

The expression of heat flux originates from the following principles.

$$\phi = S \cdot \frac{1}{\left(\frac{e_1}{\lambda_1} + \frac{e_2}{\lambda_2} + \frac{e_3}{\lambda_3} \right)} \cdot (\theta_1 - \theta_4) \quad (2-12)$$

Combining these equations, we get the overall temperature difference across all the walls:

Thermal Resistance in Series

The total thermal resistance R_{total} of the series of walls is the sum of the individual resistances:

$$R_{\text{total}} = R_1 + R_2 + R_3$$

Where:

$$\phi = \frac{1}{\left(\frac{e_1}{\lambda_1 S} + \frac{e_2}{\lambda_2 S} + \frac{e_3}{\lambda_3 S} \right)} \cdot (\theta_1 - \theta_4) = \frac{1}{(R_1 + R_2 + R_3)} \cdot (\theta_1 - \theta_4) \quad (2-13)$$

Thus, the total thermal resistance is:

$$\text{So with } R = R_1 + R_2 + R_3 \quad (2-14)$$

This gives the final expression for the thermal flux through the series of walls:

2.2.2 Equivalent Thermal Resistance of Walls in Series

As in electricity, the equivalent thermal resistance of walls in series is the sum of the thermal resistances of each wall.

2.2.3 Application Example

Study of Heat Losses by Conduction through Double Glazing

Double glazing consists of two glass panes separated by a layer of stationary dry air. The thickness of each pane is 3.5 mm, and the thickness of the air layer is 12 mm. The thermal conductivity of glass is $0,7 \text{ W.m}^{-1}.\text{°C}^{-1}$ and that of air $0,024 \text{ W.m}^{-1}.\text{°C}^{-1}$ over the temperature range studied. For a temperature drop of 5°C between the two external faces of the double glazing, calculate the thermal losses for a 1 m^2 window. (Note: This calculation neglects the effect of the convection coefficient on either side of each pane). Compare these thermal losses to those obtained with a single pane of glass of thickness 3.5 mm.

Calculation

1. Thermal Resistance of Each Layer:

Glass Pane:

2. Total Thermal Resistance of Double Glazing:

3. Thermal Flux through Double Glazing:

4. Thermal Flux through a Single Pane of Glass:

$$\phi = \frac{5.1}{\frac{2.3 \cdot 10^{-3}}{0,7} + \frac{12 \cdot 10^{-3}}{0,024}} = \frac{5}{2.0,005 + 0,5} \Phi \text{ double glazing} = 9,8 \text{ W}$$

Note on temperature profile (see figure)

The thermal resistance of the air blade is 100 times higher than that of each window, the temperature drop in the air will be 100 times higher than in each window,

That's to say :

$$\Theta_{\text{int}} - \Theta_{v1} = R_v \times \Phi = 0,005 \times 9,8 = 0,049 \text{ °C}$$

$$\Theta_{v1} - \Theta_{v2} = R_a \times \Phi = 0,5 \times 9,8 = 4,9 \text{ °C}$$

$$\Theta_{v2} - \Theta_{\text{ext}} = R_v \times \Phi = 0,005 \times 9,8 = 0,049 \text{ °C}$$

Let's compare the flow passing through double glazing to that passing through a single pane of glass to the same surface and the same temperature difference.

so

$$\phi_{\text{seul vitre}} d\theta = \frac{\theta_{\text{int}} - \theta_{\text{ext}}}{R_v} = \frac{\lambda_v \cdot S}{e_v} (\theta_{\text{int}} - \theta_{\text{ext}}) \quad (2-15)$$

$$\phi_{\text{seul vitre}} = \frac{0,7 \times 1}{3.5 \cdot 10^{-3}}, \Phi \text{ single pane} = 1000 \text{ W}$$

Conclusion : If we consider only heat transfer by conduction (as we will see in the next chapter, the result is modified due to heat transfer by convection), double glazing allows for a

100-fold reduction in thermal losses through the glass. This is primarily due to the very high thermal resistance of the air layer, as air has a low thermal conductivity.

2.3 Conduction through the Wall of a Cylindrical Tube

Consider a cylindrical tube (see Figure 2.4). Let r_1 be the radius of the inner wall, r_2 the radius of the outer wall, Θ_1 and Θ_2 and the respective temperatures of the inner and outer faces, and λ the average thermal conductivity of the material constituting the tube between Θ_1 and Θ_2

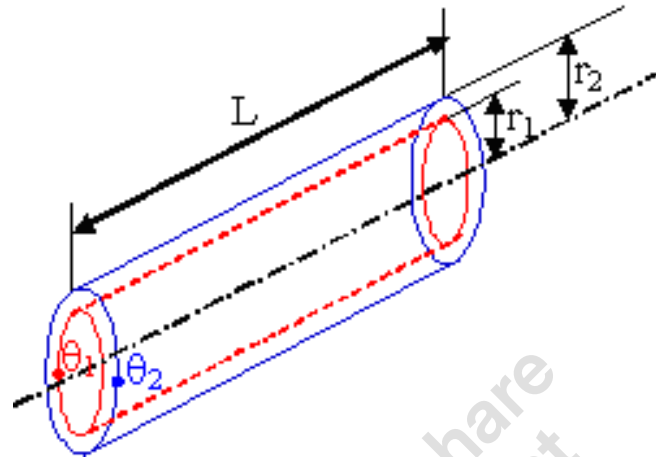


Figure 2.4. View of a cylindrical tube crossed by a conduction flow

2.3.1 Expression of the Thermal Flux of Conduction in a Cylindrical Tube

2.3.1. Expression du flux naître :

The heat flux that traverses the tube from the inside to the outside (when $\Theta_1 > \Theta_2$) for a tube length L . Due to symmetry, the heat flow lines are straight lines directed along the radii. It is said that the heat transfer is radial.

Consider a cylinder with an intermediate radius r where $r_1 < r < r_2$ and thickness dr .

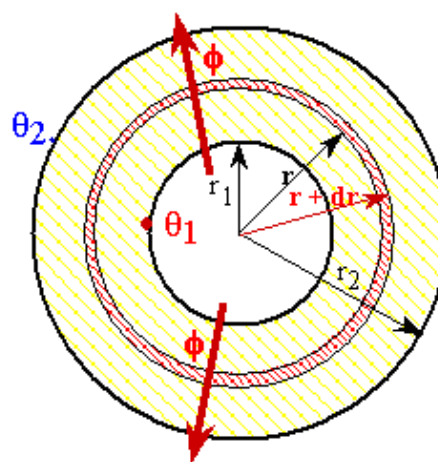


Figure 2.5. Cross-section of a hollow cylinder traversed by a conduction flux The heat flux density through this cylinder is given by Fourier's law:

$$\phi = -\lambda.S \frac{d\theta}{dr} \quad (2-16)$$

$$\text{The corresponding heat flux is: } \phi = -\lambda.S \frac{d\theta}{dr}$$

S being the area of the lateral surface of the cylinder with radius r and length L, i.e., $S=2\cdot\pi\cdot r\cdot L$

$$\text{Do : } \phi = -\lambda.2\pi.Lr \frac{d\theta}{dr} \quad d\theta = \frac{-\phi}{\lambda.2\pi.L} \frac{dr}{r} \quad \text{or else}$$

Since Φ is constant throughout the coaxial cylinder of radius r between r_1 and r_2 , the preceding equation can therefore be integrated from the inside to the outside of the cylinder as follows:"

$$\int_{\theta_1}^{\theta_2} d\theta = \frac{-\phi}{\lambda.2\pi.L} \int_{r_1}^{r_2} \frac{dr}{r} \quad (2-17)$$

$$\text{so: } (\theta_1 - \theta_2) = \frac{\phi}{\lambda.2\pi.L} (\ln r_2 - \ln r_1) = \frac{-\phi}{\lambda.2\pi.L} \ln\left(\frac{r_2}{r_1}\right) \quad (2-18)$$

$$r_{ml} = \frac{(r_1 - r_2)}{\ln\left(\frac{r_2}{r_1}\right)} = \frac{e}{\ln\left(\frac{r_2}{r_1}\right)} \quad (2-19)$$

We deduce the expression for the heat flux."

$$\phi = \frac{\lambda.2\pi.L.}{\ln\left(\frac{r_2}{r_1}\right)} (\theta_1 - \theta_2) \quad (2-20)$$

Remark: This flux does not depend on the absolute dimensions of the tube. It only depends on the ratio r_2 / r_1 . Let's transform this expression to make it similar to that of a flat wall [1]. To do this, let's recall the definition of the logarithmic mean applied to the two radii r_1 and r_2 ."

$$r_{ml} = \frac{(r_1 - r_2)}{\ln\left(\frac{r_2}{r_1}\right)} = \frac{e}{\ln\left(\frac{r_2}{r_1}\right)} \quad (2-21)$$

The difference between the radii is the thickness of the tube, denoted as e. Hence..:

$$\ln\left(\frac{r_2}{r_1}\right) = \frac{e}{r_{ml}}$$

, by substituting this into the expression for the flux

$$\phi = \frac{\lambda.2\pi.L.}{\ln\left(\frac{r_2}{r_1}\right)} (\theta_1 - \theta_2) = \frac{\lambda.2\pi.L.r_{ml}}{e} (\theta_1 - \theta_2) \quad (2-23)$$

The internal lateral surface area of the tube is: $S_1 = 2 \Pi. r_1 .L$. The external lateral surface area of the tube is: $S_2 = 2 \Pi. r_2 .L$ The logarithmic mean of these two surfaces is

$$S_{ml} = \frac{S_2 - S_1}{\ln\left(\frac{S_2}{S_1}\right)} = \frac{2\pi.L.(r_2 - r_1)}{\ln\left(\frac{2\pi.L.r_2}{2\pi.L.r_1}\right)} = 2\pi.L.\frac{(r_2 - r_1)}{\ln\left(\frac{r_2}{r_1}\right)} = 2\pi.L.r_{m,l} \quad (2-24)$$

Hence, the final expression for the flux through a tube is:"

$$\phi = \frac{\lambda.S_{ml}}{e}(\theta_1 - \theta_2) \quad (2-25)$$

This expression is similar to the one obtained for a flat wall, where the surface S is replaced by the logarithmic mean surface. Very often, since the thickness of the tube is small, we can replace the logarithmic mean surface with the arithmetic mean surface, which is...

$$S_{ma} = (S_1 + S_2) / 2 = \Pi .L. (r_1 + r_2) \quad (2-26)$$

$$S_{ma} = (S_1 + S_2) / 2 = \Pi .L. (r_1 + r_2) \quad (2-26)$$

2.3.2. Expression of the thermal resistance of a cylindrical tube

From the expression of the heat flux, we derive the expression of the thermal resistance of

$$\text{tube: } R = \frac{\theta_1 - \theta_2}{\phi} = \frac{\theta_1 - \theta_2}{\frac{\lambda.S_{ml}}{e}(\theta_1 - \theta_2)} \text{ donc : } R = \frac{e}{\lambda.S_{ml}} \quad (2-27)$$

$$\text{so } R = \frac{e}{\lambda.S_{ml}} = \frac{r_1 - r_2}{\lambda .2\pi.L \frac{(r_1 - r_2)}{\ln\left(\frac{r_2}{r_1}\right)}} = \frac{\ln\left(\frac{r_2}{r_1}\right)}{\lambda .2\pi.L} \quad (2-28)$$

2.3.3 Application example

Consider a steel tube 20/27, where the temperature of the inner wall is $\Theta_1 = 119,75^\circ\text{C}$ and the temperature of the outer wall is $\Theta_2 = 119,64^\circ\text{C}$.

The thermal conductivity of steel is $\lambda = 46 \text{ W.m}^{-1} . ^\circ\text{C}^{-1}$

a) Thermal Resistance of the Tube for a Length of 1 m

$$R = \frac{e}{\lambda.S_{ml}} = \frac{r_1 - r_2}{\lambda .2\pi.L \frac{(r_1 - r_2)}{\ln\left(\frac{r_2}{r_1}\right)}} = \frac{\ln\left(\frac{r_2}{r_1}\right)}{\lambda .2\pi.L}, \text{ d'ou } R = \frac{\ln\left(\frac{27/2}{20/2}\right)}{46.2\pi.1} \quad (2-29)$$

So $R = 1,038.10^{-3} \text{ } ^\circ\text{C.W}^{-1}$

b) Corresponding Thermal Flux

The thermal flux is given by: b) Le flux traversant par conduction un tube de 1m de longueur est:

$$\phi = \frac{\theta_1 - \theta_2}{R_{\text{totale}}} = \frac{119.75 - 119.64}{1.038.10^{-3}} \quad (2-30)$$

so $\Phi = 105,97 \text{ W}$

2.4 Conduction through Two Concentric Tubes in Perfect Thermal Contact

Consider two concentric tubes of length (L) in perfect thermal contact (see Figure 2.6). θ_1 is the temperature of the inner surface of tube 1 with thermal conductivity λ_1 , θ_3 is the temperature of the outer surface of tube 2 with thermal conductivity λ_2 . θ_2 is the temperature at the interface between the two tubes.

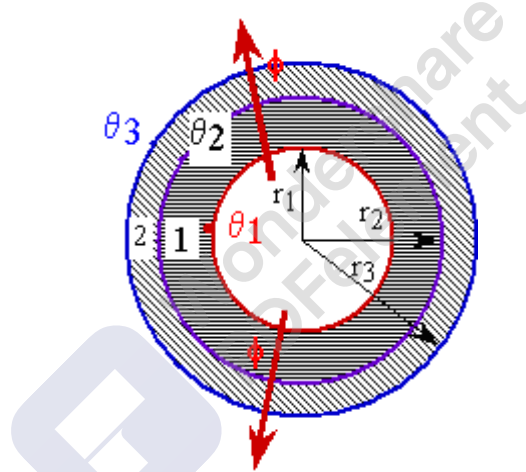


Figure 2.6 Cross-Sectional View of Two Concentric Cylindrical Tubes in Contact

2.4.1 Expression of the Thermal Flux through Concentric Tubes

To derive the expression for the thermal flux through the concentric tubes, we need to consider the thermal resistances of both tubes and the interface temperature θ_2 .

2.4.1 Expression of the Equivalent Thermal Resistance of Two Concentric Cylindrical Tubes

Tube 1 constitutes a first thermal resistance R_1 to heat transfer:

$$R_1 = \frac{e_1}{\lambda_1 S_{1ml}} \quad (2-31)$$

where e_1 is the thickness of tube 1 and S_{1ml} is the logarithmic mean area of tube 1.

Tube 2 constitutes a second thermal resistance R_2 to heat transfer:

$$R_2 = \frac{e_2}{\lambda_2 S_{2ml}} \quad (2-32)$$

where e_2 is the thickness of tube 2 and S_2 is the logarithmic mean area of tube 2.

These two resistances are in series, and the equivalent resistance is:

$$R = R_1 + R_2 = \frac{e_1}{\lambda_1 S_{1ml}} + \frac{e_2}{\lambda_2 S_{2ml}} \quad (2-33)$$

2.4.2 Expression of the Thermal Flux through Two Concentric Cylindrical Tubes

Based on the equivalent resistance calculated above, the expression for the thermal flux (if $\Theta_1 > \Theta_3$) is:

$$\phi = \frac{\theta_1 - \theta_3}{R_1 + R_2} = \frac{\theta_1 - \theta_3}{\frac{e_1}{\lambda_1 S_{1ml}} + \frac{e_2}{\lambda_2 S_{2ml}}} \quad (2-34)$$

This calculation is used to determine the effect of tube insulation or to predict the increase in thermal resistance when a tube is fouled or scaled.

2.4.3 Application Example

The interior of the 20/27 tube studied in the previous example (2.3.3) is scaled with a thickness of 2 mm. Assume that the internal and external temperatures remain unchanged: the temperature of the inner wall is $\Theta_1 = 119,75^\circ\text{C}$ and that of the outer wall is $\Theta_2 = 119,64^\circ$

Calculate:

1. Calculate the natural logarithm:
2. Calculate the denominator:
3. Calculate the thermal resistance:
 - a) The thermal resistance of the scale layer R_t for a length of 1 m

Avec $r_t = r_1$ – épaisseur couche de tartre

- Thickness of the scale layer: $e = 0.002$ m

The thermal resistance R_t of the scale layer is given by:

$$\phi = \frac{e_t}{\lambda_t S_{ml}} = \frac{r_1 - r_t}{\lambda_t \cdot 2\pi \cdot L \frac{(r_1 - r_t)}{\ln\left(\frac{r_1}{r_t}\right)}} = \frac{\ln\left(\frac{r_1}{r_t}\right)}{\lambda_t \cdot 2\pi \cdot L} \quad (2-35)$$

From the previous example (2.3.3), the thermal resistance of the steel tube for a length of 1 m is

$$R_t = \frac{\ln\left(\frac{10}{8}\right)}{2,2 \cdot 2\pi \cdot 1} \quad (2-36)$$

$$\text{So } R_t = 1,614 \cdot 10^{-2} \text{ } ^\circ\text{C} \cdot \text{W}^{-1}$$

- Thermal conductivity of the scale:

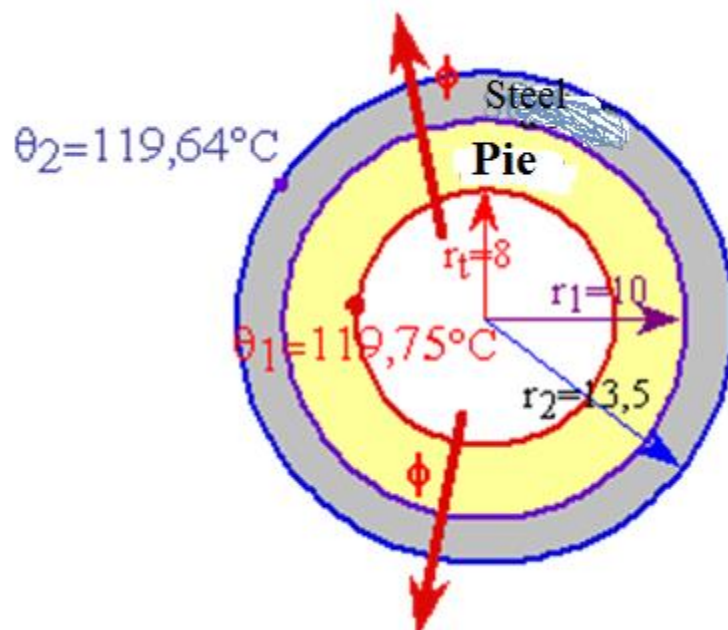
- Length of the tube: $L = 1 \text{ m}$

$$: R_{\text{total}} = R_{\text{steel tube}} + R_{\text{tartar}} = 1,038 \cdot 10^{-3} + 1,614 \cdot 10^{-2} = 1,718 \cdot 10^{-2} \text{ } ^\circ\text{C} \cdot \text{W}^{-1}$$

c) c) The corresponding thermal flux

$$\phi = \frac{\theta_1 - \theta_2}{R_{\text{totale}}} = \frac{119,75 - 119,64}{1,718 \cdot 10^{-2}} \quad (2-37)$$

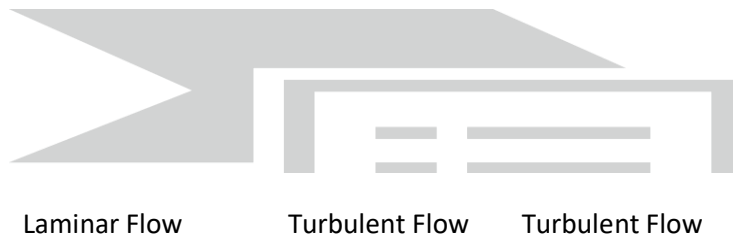
$$\text{So } \Phi = 6,4 \text{ W}$$



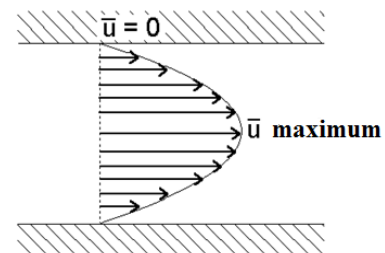
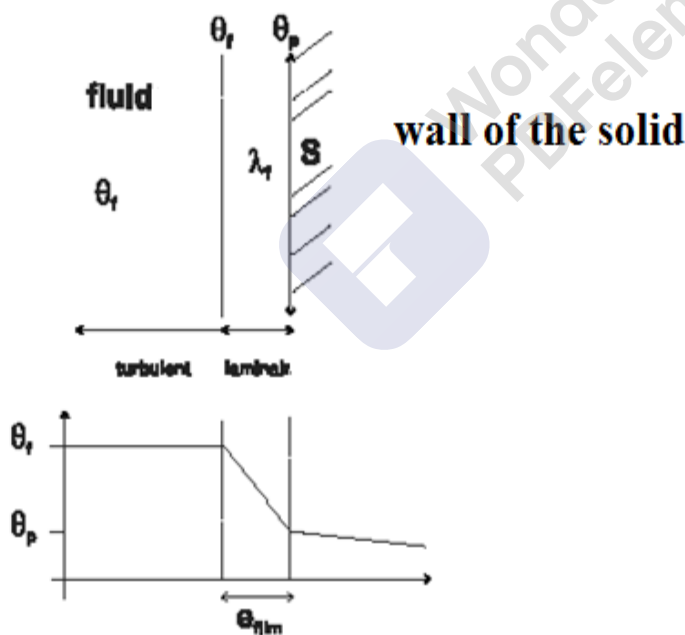
3. Transmission de chaleur par convection

La convection est le mode de propagation de la chaleur dans un fluide (liquide ou gazeux) en mouvement. La transmission de la chaleur entre un solide et un fluide s'effectue par l'action combinée de la conduction et de la convection. Ce mécanisme de transfert est régi par la loi de Newton [2]:

3. Heat Transfer by Convection



Convection is the mode of heat propagation in a moving fluid (liquid or gas). Heat transfer between a solid and a fluid occurs through the combined action of conduction and convection. This transfer mechanism is governed by Newton's law [2]:



Heat Transfer in a Stationary Fluid or Laminar Flow

In a stationary fluid or laminar flow, it is the emergence of temperature differences that triggers convection. In turbulent flow in contact with a solid,

there exists along the wall a thin layer of fluid in laminar flow where heat propagation occurs by conduction. This layer hinders the quality of heat transfer and should be as thin as possible, which leads to a high flow velocity.

It is assumed that within the boundary layer there is no mixing of matter and that heat is transmitted by conduction perpendicular to the wall. Since the conductivity of fluids is low compared to that of solids, this layer constitutes a zone of resistance to heat transfer.

Therefore, there is a significant temperature variation within this layer. This explains why the temperature of a heat exchanger wall can be substantially different from the temperature measured within the fluid.

Within the fluid, heat is transmitted effectively through mixing, and the temperature becomes perfectly homogeneous. This temperature is called the fluid temperature or the fluid mixing temperature.

We conclude from this study that the phenomenon of convection is "reduced," from a thermal perspective, to conduction within a thin layer. The heat flux exchanged by convection between the fluid and the wall can thus be written as:

$$\Phi = \frac{\lambda_f \cdot S}{e_{\text{film}}} \cdot (\theta_f - \theta_p) \quad (3-1)$$

où λ_f is the thermal conductivity of the fluid, est la conductivité thermique du fluide

e_{film} : is the thickness of the film,

S : is the surface area of the wall,

θ_f et θ_p are the temperatures of the fluid and the wall, respectively.

Unfortunately, the thickness of the thin layer e_{film} is rarely known precisely because it depends on numerous factors, λ_f varies with temperature, and the temperature itself varies within the layer. For these reasons, in heat transfer by convection, the heat flux is often expressed in the following form:

$$\Phi = hS (T_p - T_\infty) \quad (3-2)$$

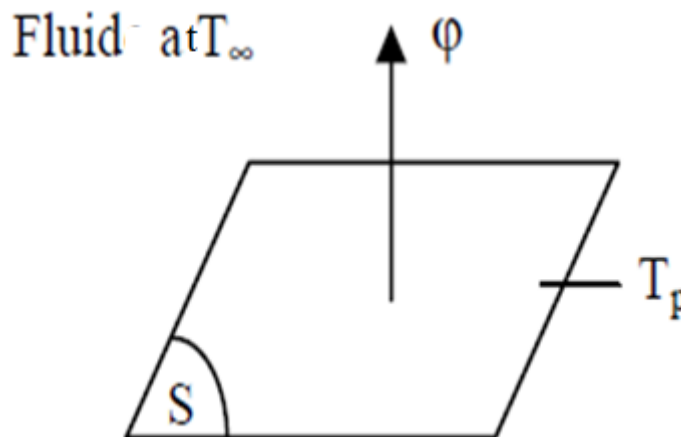


Figure 1.4: Schematic of convective heat transfer

where h is the thermal convection coefficient. It has the same dimension as λ .

The thermal resistance of heat transfer by convection R is therefore equal to:

$$R = \frac{1}{h \cdot S} \quad (3-3)$$

Where:

- Φ is the heat flux transmitted by convection (W),
- h is the convective heat transfer coefficient ($\text{W}/\text{m}^2 \cdot ^\circ\text{C}$),
- T_p is the surface temperature of the solid ($^\circ\text{C}$),
- T_∞ is the temperature of the fluid far from the solid surface ($^\circ\text{C}$),
- SSS is the surface area of the solid/fluid contact (m^2).

Note: The value of the convective heat transfer coefficient h depends on the nature of the fluid, its temperature, velocity, and the geometric characteristics of the solid/fluid contact surface.

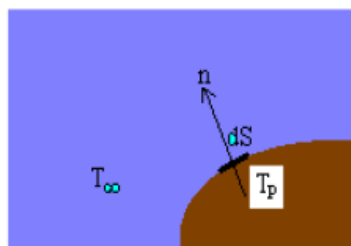
1. Introduction and Generalities
2. Thermal Conduction
3. Heat Equation
4. Thermal Radiation
5. Thermal Convection
6. Heat Exchangers
7. Small Application Class

3.1 Forced Convection without Phase Change

Determining the coefficient h depends on:

- the conduction between fluid particles,
- the mixing of these particles due to overall fluid motion,
- heat exchange that may involve a phase change.

3.1.1 Convective Heat Transfer Coefficient



d^2Q : Quantity of heat crossing dS during time dt , in Joules

$\frac{d}{dt}(dQ)$ Heat Flux, in Watts

$$d^2Q = h (T_p - T_\infty) dS \cdot dt \text{ en } W/(m^2 \cdot K) \quad (3-3)$$

3.1.2 Determination of Convective Heat Transfer Coefficient

The challenge in convection is indeed to determine this coefficient based on fluid flow conditions, geometric characteristics of the walls, and potential phase changes of the fluid. Experimentation often provides the most valuable information regarding the value of these coefficients.

Determining the coefficient h depends on:

- Conduction between fluid particles,
- Mixing of these particles due to overall fluid motion,
- Heat exchange that may involve a phase change.

3.2 Different Convective Exchanges

- Monophasic heat exchange in forced convection
- Monophasic heat exchange in natural convection
- Heat exchange accompanied by boiling
- Heat exchange accompanied by condensation

3.3 Forced Convection without Phase Change

The issue is to specify the expression of the thermal flux Φ exchanged between the external fluid at temperature T_∞ and a unit length of the pipe surface at temperature T_p

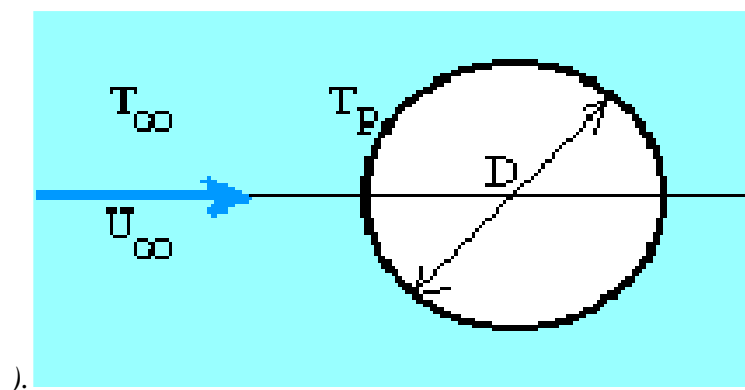


Figure 3.3. Flow around a cylindrical

Flow transferred in Watt

$$\Phi = h(T_p - T_\infty)S \quad (3-4)$$

3.3.1 Nusselt Number Nu

$$Nu = \frac{h D}{\lambda} \quad (3-5)$$

Significance of the Nusselt Number Nu :

Nu is the dimensionless form of the convective heat transfer coefficient h

$$F_{convecté} = h (T_p - T_\infty) (DL) \quad (3-6)$$

$$\text{Reference flux} = \text{conduction flux:} \quad = \lambda(DL) [(T_p - T_\infty) / D] \quad (3-7)$$

where:

h is the convective heat transfer coefficient,

L is the characteristic length,

k is the thermal conductivity of the fluid,

T_p is the temperature of the pipe surface

T_∞ is the temperature of the fluid far from the surface

$$Nu = \frac{F_{convecté}}{\text{Flux de référence}} = \frac{h (T_p - T_\infty)(DL)}{\lambda (DL)[(T_p - T_\infty)]/D} = \frac{h D}{\lambda} \quad (3-8)$$

2.3.2 Reynolds Number Re

The Reynolds Number Re is a dimensionless quantity used to predict flow patterns in different fluid flow situations. It is defined as:

$$Re = \frac{\rho U_\infty D}{\mu} \quad (3-9)$$

where:

ρ is the fluid density (kg/m³),

v is the flow velocity (m/s),

L is the characteristic length (m),

μ is the dynamic viscosity of the fluid (Pa·s or N·s/m²).

The significance of the Reynolds Number Re lies in its ability to indicate whether the flow will be laminar or turbulent:

- For $Re < 2000$, the flow is typically laminar.
- For $2000 < Re < 4000$, the flow is in the transition region.
- For $Re > 4000$, the flow is typically turbulent.

The significance of the Reynolds Number Re

$$Re = \frac{\text{Force d'inertie}}{\text{Force de viscosité}} = \frac{\rho U_{\infty} D}{\mu} \quad (3-10)$$

Reynolds number Re is a dimensionless quantity used to characterize the flow regime of a fluid.

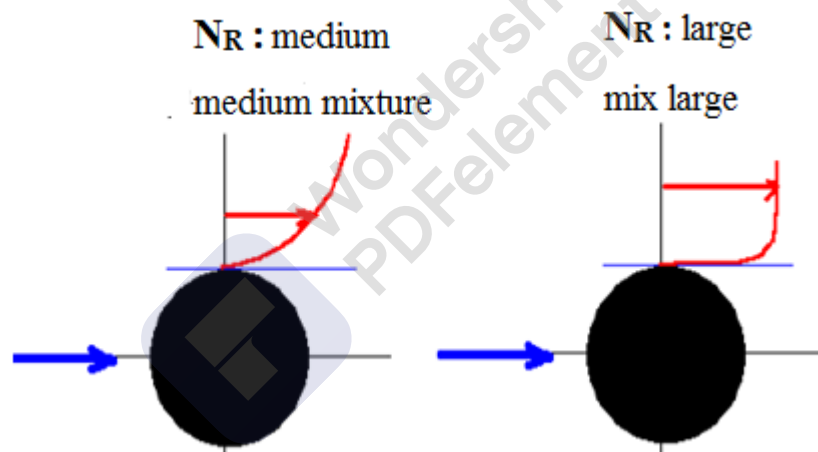


Figure 3.4 Efficiency of flow regime around a cylindrical tube

3.3.3 Nombre de Prandtl Pr

$$Pr = \frac{\mu C}{\lambda} \quad (3-11)$$

Meaning of the Prandtl Pr Number

dynamic	viscosity	thermal	diffusivity
$Pr = \frac{\text{Viscosité dynamique}}{\text{Diffusivité thermique}} = \frac{\mu/\rho}{\lambda/\rho C} = \frac{\mu C}{\lambda} \quad (3-12)$			

Influence de la diffusivité thermique a

Pr: compares the respective influences: the fluid velocity profile (viscosity) the temperature profile (diffusivity) For usual gases, Pr is close to 0.75 Influence of thermal diffusivity a

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{a} \cdot \frac{\partial T}{\partial t} \quad \text{with} \quad a = \frac{\lambda}{\rho C} \quad (3-13)$$

dT proportional to a

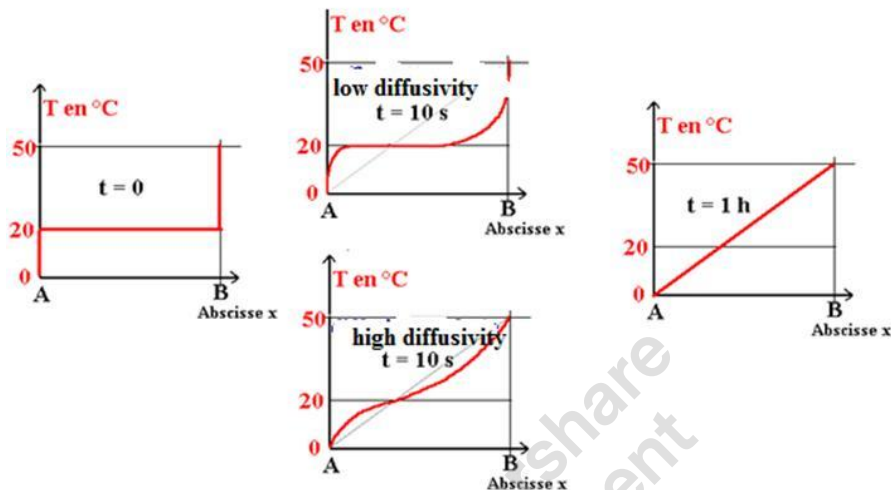


Figure 3.5. Efficacité de diffusivité thermique[2]

2.3.4 Conclusion de l'analyse dimensionnelle

Le transfert de chaleur convectif implique une relation entre 4 nombres sans dimension

Figure 3.5. Thermal diffusivity efficiency[2]

2.3.4 Conclusion of the dimensional analysis Convective heat transfer involves a relationship between 4 dimensionless numbers:

$F(\text{Nu}, \text{Re}, \text{Pr}, \text{Ec}) = 0$

$$\text{Nu} = \frac{h D}{\lambda} \quad (3-14)$$

$$\text{Re} = \frac{\rho \cdot U_{\infty} \cdot D}{\mu} \quad (3-15)$$

$$\text{Pr} = \frac{\mu \cdot C}{\lambda} \quad (3-16)$$

2.4 Derived numbers

2.4.1 Peclet number: ratio of heat fluxes by convection and by conduction •

$$Pe = Re \cdot Pr = \frac{\rho \cdot U \cdot D}{\mu} \cdot \frac{\mu \cdot C}{\lambda}$$

$$Pe = \frac{U \cdot D}{a}$$
(3-17)

With a, thermal diffusivity:

$$a = \frac{\lambda}{\rho \cdot C_p}$$
(3-18)

There are also the numbers of Stanton, Grashof, Froude, Weber, Rayleighp

3.5 Law of convection forced

$$F(Nu, Re, Pr) = 0 \text{ ou } Nu = F(Re, Pr)$$
(3-19)

$$\frac{h D}{\lambda} = f\left(\frac{\rho \cdot U_{\infty} \cdot D}{\mu} \cdot \frac{\mu \cdot C}{\lambda}\right)$$
(3-20)

3.5.1 Flow in a tube

Permanent regime in a circular cylindrical pipe of internal diameter D . •Heat flow $D\phi$ exchanged through the lateral wall area dS between the abscissa x and $x+dx$:

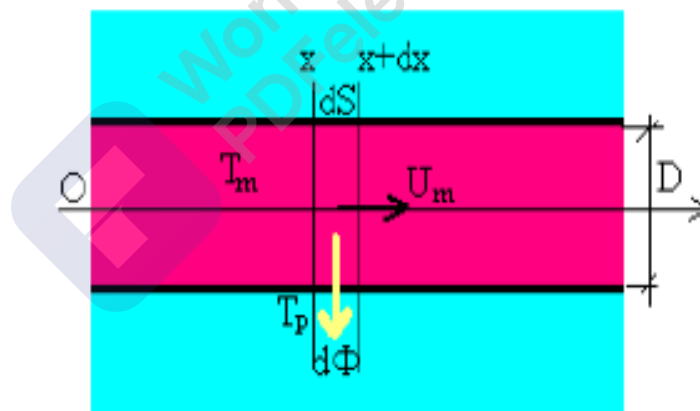


Figure 3.6. Flow inside a cylindrical tube

$$d\Phi = h (T_m - T_p) (\pi D dx)$$
(3-21)

Experience shows that the internal convection coefficient h_i in a section depends on the following seven quantities:

u_m : average fluid speed

ρ : Density of the fluid

C_p : specific heat of the fluid

μ : dynamic viscosity of the fluid

λ : Thermal conductivity of the fluid

D_i : inside diameter of the tube

X : abscissa of the section considered with the origin placed at the entrance of the tube.

Determining the coefficient h_i by experiment is impossible to achieve because of too many experiments required. Dimensional analysis can significantly simplify this problem. It shows that there exists a function F with three variables verifying the relationship:

$$\frac{h_i \cdot D_i}{\lambda} = F\left(\frac{\rho \cdot U_m \cdot D_i}{\mu}, \frac{C_p \cdot \mu}{\lambda}, \frac{x}{D_i}\right) \quad (3-22)$$

We therefore define four dimensionless numbers:

$$\text{Nusselt number: } Nu = \frac{h_i \cdot D_i}{\lambda} \quad (3-23)$$

$$\text{Reynolds number: } Re = \frac{\rho \cdot U_m \cdot D_i}{\mu} \quad (3-24)$$

$$\text{Prandtl number: } Pr = \frac{C_p \cdot \mu}{\lambda} \quad (3-25)$$

$$\frac{x}{D_i} \quad (3-26)$$

The Nusselt, Prandtl and Reynolds numbers respectively characterize the heat exchange, the thermal properties of the fluid and the flow regime of the fluid. The number x/D_i is the term representative of edge effects; it no longer occurs when we are far enough from one of the ends of the tube.

The experiment is then used to determine the function F , that is to say a mathematical correlation linking these numbers.

2.5.2 Exchange coefficient in turbulent regime

In the case of a smooth tube with permanent flow, we use the Colburn relation:

$$: Nu = 0,023 Pr^{1/3} Re^{0.8} \quad (3-27)$$

Valid for $10000 < Re < 120000$, $0.7 < Pr < 120$, $L/D_i > 60$. The numerical values vary depending on the nature of the fluids

Conditions of application:

- The flow regime must be perfectly established $x/D > 60$

- $0.7 < \text{Pr} < 100$.

Turbulent regime not established

 $x/D < 60$

$$Nu = 0,023 \text{ Pr}^{1/3} \text{ Re}^{0.8} (1 + (\frac{D}{X})^{0.7}) \quad (3-28)$$

3.5.3 Laminar regime $Re < 2000$, •experimental correlations of L  v  que, with:

$$A = \frac{1}{\text{Re. Pr}} \frac{x}{D} = \frac{V \cdot D}{\alpha} \quad (3-29)$$

$$\alpha = \frac{\lambda}{\rho \cdot C_v} \quad (3-30)$$

$$\text{Nu} = 3.66 \quad \text{pour } A \succ 0.05$$
$$\text{Nu} = 1.06 A^{0.4} \quad \text{pour } A \prec 0.05$$

3.5.4 Application example

• Pipe diameter $D = 20 \text{ mm}$

Flow rate $Q = 0.5$ l/s of water at 50°C

•Determine the heat flux transmitted by convection of the fluid towards the wall, per linear meter of pipe, within the framework of the following hypotheses:

- Constant water inlet temperature;

- Quite thin tube wall - conduction is neglected;

–Outside temperature = 15°C ;

- Perfectly established flow

- Physical properties of water:

–Density at 50°C: $\rho = 988 \text{ kg/m}^3$

–Dynamic viscosity at 50°C: $\mu = 0.55 \cdot 10^{-3} \text{Pa.s}$

–Thermal conductivity at 50°C: $\lambda = 0.639 \text{ W/(m.}^\circ\text{C)}$

– Specific thermal capacity at 50°C: $C_p = 4184 \text{ J / (kg. } ^\circ\text{C)}$

3.5.5 Solving a forced convection problem

1- a geometry

2- a characteristic dimension L

3- The gap $T_p - T_\infty$ between wall and fluid

4- The speed U_∞ of the fluid

5- ρ , μ , Ce λ of the fluid

Example: A pipe with a circular section of diameter $D = 20$ mm carrying hot water.

1-a geometry

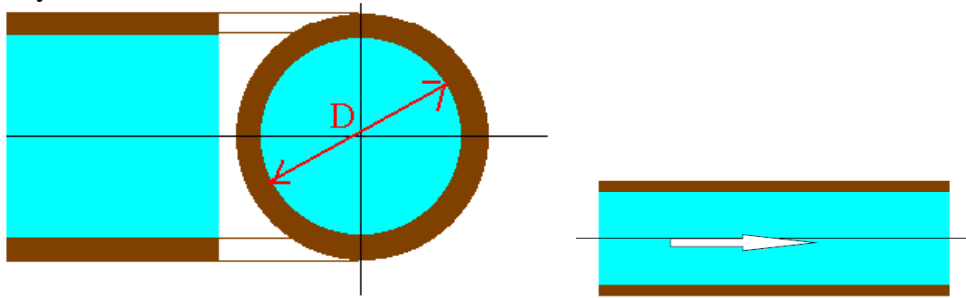


Figure 3.6. Flow inside a cylindrical tube 2- a characteristic dimension L 3-The temperature difference $T_p - T_\infty$ between wall and fluid

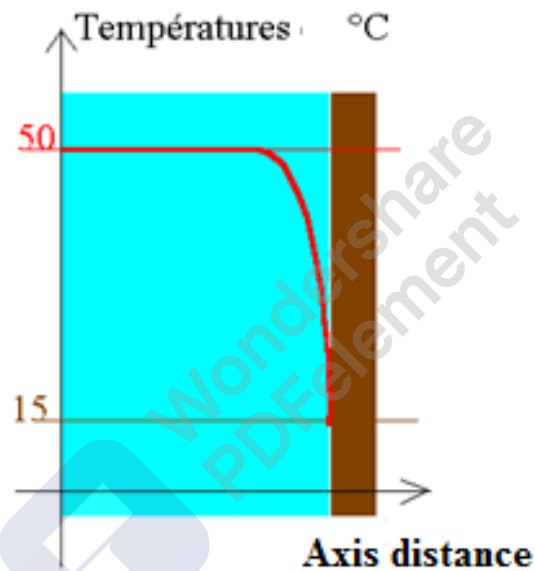
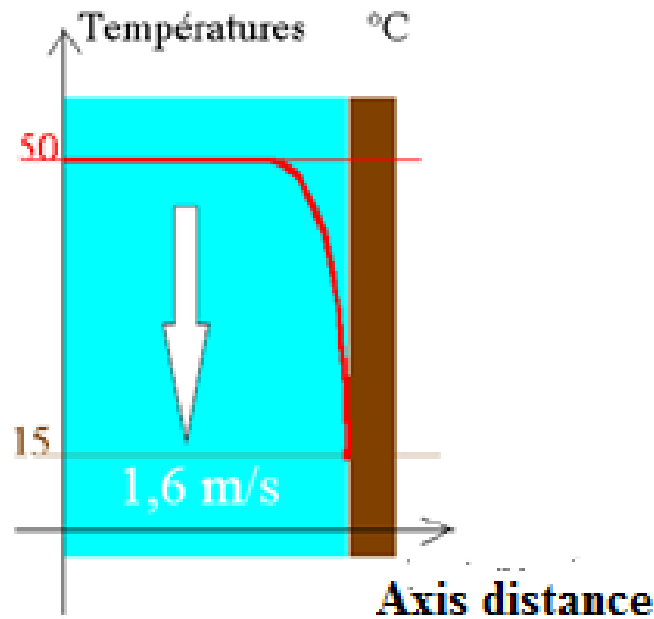


Figure 3.7. Temperature distribution inside a cylindrical tube

Example: The pipe transports water at the average temperature: $T_m = 50^\circ\text{C}$ while the wall is at the temperature: $T_p = 15^\circ\text{C}$

4- The speed U_∞ of the fluid

The pipe carries a flow: $Q = 0.5$ l/s The average speed of the flow is then: $U_m = Q/S = 1.6$ m/s



5- ρ , μ , C and λ of the fluid

For water:

Density at 50°C: $\rho = 988 \text{ kg/m}^3$

Dynamic viscosity at 50°C: $\mu = 0.55 \cdot 10^{-3} \text{ Pa.s}$

Thermal conductivity at 50°C: $\lambda = 0.639 \text{ W/(m.}^\circ\text{C)}$

Specific heat capacity at 50°C: $C = 4184 \text{ J/(kg.}^\circ\text{C)}$

Calculation of the convective transfer coefficient h

$$\frac{h \cdot D}{\lambda} = f\left(\frac{\rho \cdot U_\infty \cdot D}{\mu}, \frac{\mu \cdot C}{\lambda}\right) \quad (3-31)$$

1 - Calculation of the Prandtl Number of the fluid

$$\text{Pr} = \frac{\mu \cdot C}{\lambda} = \frac{0,55 \cdot 10^{-3} \cdot 4184}{0,639} = 3.60 \quad (3-32)$$

2 - Calculation of the Reynolds Number of the fluid

$$\text{Re} = \frac{\rho \cdot U_m \cdot D}{\mu} = \frac{988 \cdot 1,59 \cdot 0,02}{0,55 \cdot 10^{-3}} = 57124 \quad (3-33)$$

For: $104 < \text{Re} < 1.2 \times 10^5$ and: $0.7 < \text{Pr} < 100$

we apply the COLBURN correlation

$$Nu = 0,023 Pr^{1/3} Re^{0.8} \quad (3-34)$$

Calculation of the Nusselt Number (Colburn Formula)

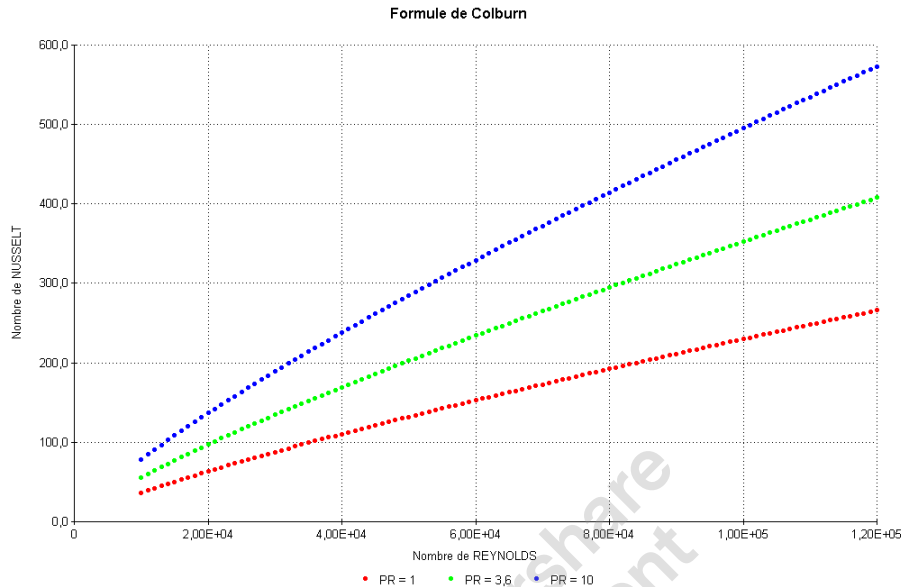


Figure3.8. Nusselt number calculation diagram (Colburn formula) [2]

5.6 Calculation of the heat flux transmitted by convection

Calculation of the heat flux transmitted by convection

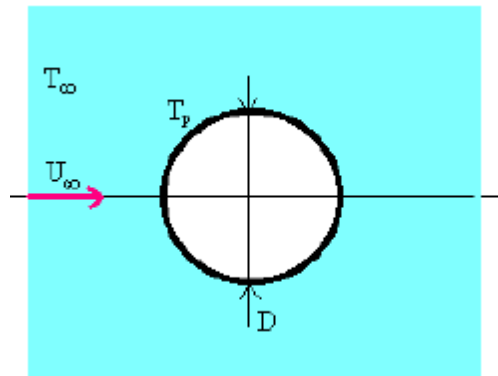
$$Nu = 224 = \frac{h D}{\lambda} \rightarrow h = \frac{\lambda Nu}{D} = \frac{0,639.224}{0,02} = 7156 \text{ (W / m}^2 \cdot \text{°C)} \quad (3-35)$$

Calculation of the heat flux transmitted by convection

$$d\Phi = h(T_p - T_\infty) \pi \cdot D dx \cdot (dS) \quad (3-36)$$

$$W = \frac{d\Phi}{dx} = h(T_m - T_p) \pi \cdot D = 15,7 \text{ kW / m} \quad (3-37)$$

3.6 Flow around a tube



$$\text{For a gas: } Nu = A \cdot Re^m \quad (3-38)$$

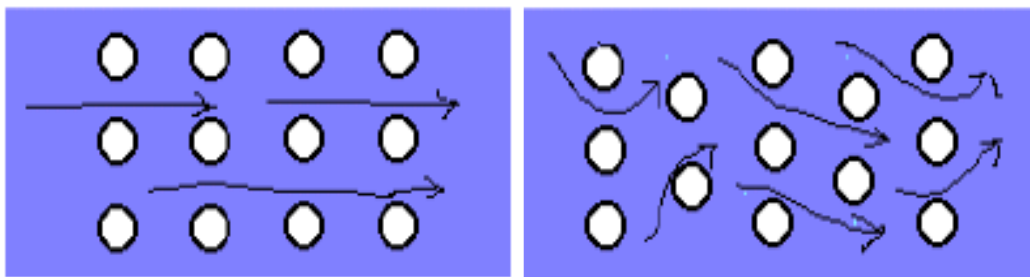
$$\text{For a liquid: } Nu = 1,11 \cdot A \cdot Pr^{0,31} Re^m \quad (3-39)$$

Tableau 3.1 Characteristic values of coefficients

Re	A	m
$1 < Re < 4$	0.891	0.330
$4 < Re < 40$	0.821	0.385
$40 < Re < 4 \cdot 10^3$	0.615	0.466
$4 \cdot 10^3 < Re < 4 \cdot 10^4$	0.174	0.618
$4 \cdot 10^4 < Re < 4 \cdot 10^5$	0.024	0.805

3.6.1 Case of tube exchangers

It is shown that depending on the type of tube bundle, the external convection coefficient h_e is different. It is higher when turbulence increases



3.6.2 Application exercise

- Calculate the length of tube needed for an air-water exchanger
- Temperatures
- Air in = 800 °C

- Air out = 40°C
- Water out = 40°C
- Average power supplied = 10 kW
- Tube diameter = 10 mm

Aligned beam

$$Nu = B \cdot (Re)^{0.66} \cdot (Pr)^{0.33} \quad (2-40)$$

Beam aligned: B=0.26

Staggered beam: B=0

$$\frac{d\Phi}{dS} = \frac{d^2Q}{dS} = -\lambda \cdot \left(\frac{\partial T}{\partial n} \right)_{n=0} \quad (3-41)$$

$$d^2Qh(T_p - T_\infty)dSdT \quad (3-42)$$

$$h = -\frac{\lambda}{(T_p - T_\infty)} \cdot \left(\frac{\partial T}{\partial n} \right)_{n=0} [W / m^2 \cdot ^\circ k] \quad (3-43)$$

3.7 Flow along a plate

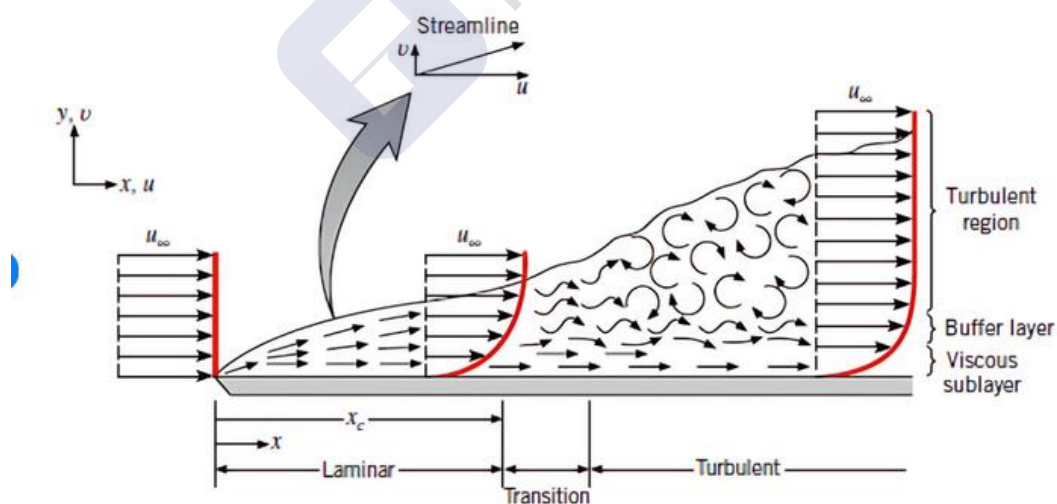


Figure 3.11 Flow along a plate

$$\frac{d\Phi}{dS} = \frac{d^2Q}{dS} = -\lambda \cdot \left(\frac{\partial T}{\partial n} \right)_{n=0} \quad (3-44)$$

$$d^2 Q h (T_p - T_\infty) dS dT$$

$$h = -\frac{\lambda}{(T_p - T_\infty)} \cdot \left(\frac{\partial T}{\partial n} \right)_{n=0} \left[W / m^2 \cdot ^\circ k \right] \quad (3-45)$$

3.7.1 Case of a flat wall – Laminar regime

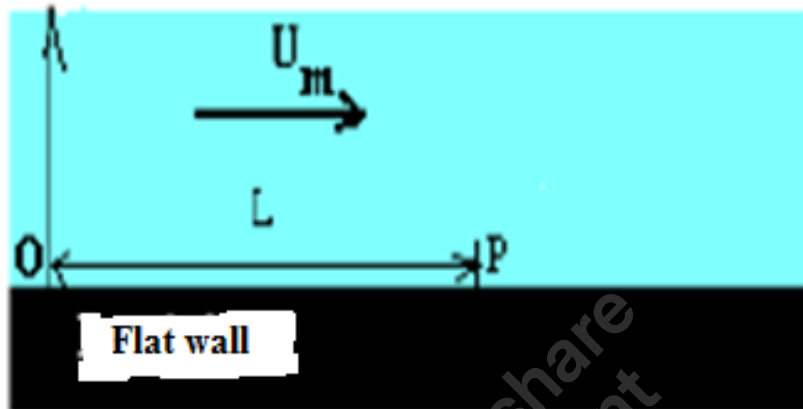


Figure 3.12. Flow along a plate

$$Re = \frac{\rho U_m L}{\mu} \quad (3-46)$$

$$Nu_L = \frac{h L}{\lambda} \quad (3-47)$$

Laminar speed $Re < 2000$

$$Nu = \frac{2}{3} Re^{0.5} \cdot Pr^{0.33} \quad (3-48)$$

Turbulent regime:

$$Nu = 0,036(Re)^{0.8} \cdot (Pr)^{0.33} \quad (3-49)$$

3.8 Natural convection:

3.8.1 Grashof and Froude numbers

$$Gr = \frac{\alpha \cdot g \cdot D^3 \cdot \Delta T}{\gamma^2} \quad (3-50)$$

with α = isobaric volume expansion coefficient of the fluid

$$\alpha = \frac{1}{v} \left(\frac{\partial v}{\partial T} \right)_{p = cte} \quad (3-51)$$

For a perfect fluid $\alpha = \frac{1}{T}$: so

$$Gr = \frac{\Delta T}{T} \cdot \frac{g \cdot D^3}{\nu^2} \quad (3-52)$$

$$Fr = \frac{U^2}{g \cdot L} \quad (3-53)$$

Relationship between upward thrust forces due to a temperature difference and viscosity forces

3.8.2 Froude number

3.8.3 Natural convection boundary layer

3.8.3.1 Flow along a plate

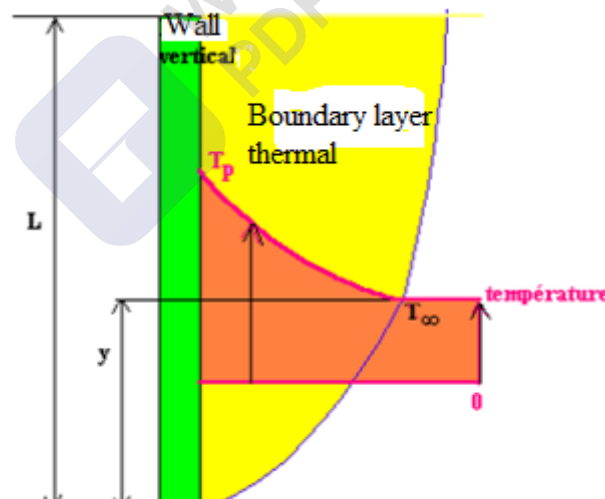


Figure 3.13. Flow along a plate

Relationship between viscosity, gravity and inertia forces.

$$Gr = \frac{\alpha \cdot g \cdot \Delta T \cdot \rho^2 \cdot L^3}{\mu^2} \quad (3-54)$$

Forces de gravité Par unité de volume

$$Gr = \frac{\alpha \cdot g \cdot \Delta T}{\left(\frac{\mu}{\rho}\right)^2 \cdot \frac{1}{L^3}} \quad (3-55)$$

8.4 Natural laminar and turbulent convection

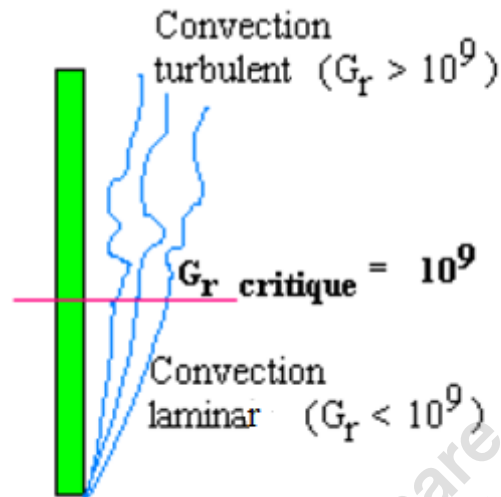


Figure 3.14. Convection regime of a flow along a plate

$$Nu = (Gr.Pr)^n \quad (3-56)$$

Calculation at average temperature, fluid-wall

Laminar: $n = 1/4$

Turbulent: $n = 1/4$

Table 3.2 Form factor C

Geometry and orientation of the wall	Characteristic dimension L	C in laminar convection	C in turbulent convection
Vertical plate	height	0.59 ($10^4 < Gr_r.P_r < 10^9$)	0.13($10^9 < Gr_r.P_r < 10^{13}$)
Horizontal cylinder	Outside diameter	0.53 ($10^3 < Gr_r.P_r < 10^9$)	0.10($10^9 < Gr_r.P_r < 10^{13}$)
Horizontal plate	width	0.54 ($10^5 < Gr_r.P_r < 2.10^7$)	0.14 ($2.10^7 < Gr_r.P_r < 3.10^{13}$)
Heated upwards	width	0.27 ($3.10^5 < Gr_r.P_r < 3.10^{10}$)	0.07 ($3.10^3 < Gr_r.P_r < 10^{13}$)

3.8.5 Application example: sunny wall

$$T_p = 313^\circ\text{K}$$

$$T_a = 293^\circ\text{K}$$

$$T_m = 303^\circ\text{K}$$

$$\rho = 1,149 \text{ kg/m}^3$$

$$\lambda = 0.0258 \text{ W/(m.K)}$$

$$\mu = 18.410 \cdot 10^{-6} \text{ Pa.s}$$

$$C_p = 1006 \text{ J/(kg.K)}$$

$$\text{Pr} = 0.72$$

$$\text{Gr} = 5.61 \cdot 10^{11}$$

$$\text{Ra} = 4.02 \cdot 10^{11}$$

$$\text{Nu} = 0.13 \text{Ra}^{0.33} = 960$$

$$h = \frac{\text{Nu}_L \cdot \lambda}{L} = 4.13 \text{ W/M}^2\text{K} \quad (3-57)$$

3.9 Transfer in a tubular exchanger.

We have two concentric cylindrical tubes of radius r_i and r_e and of length L . A hot liquid at temperature θ_i circulates in the inner tube and a cold liquid at temperature θ_e circulates in the annular space. The overall transfer of heat from hot liquid to cold liquid takes place in three phases:

Convection in the inner tube (h_i) of the hot liquid at the inner wall of the inner tube

Conduction in the inner tube wall (λ)

Convection in the annular space of the outer wall of the inner tube to the cold liquid (h_e)

Using the additivity property of thermal resistances in series, we can deduce the value of the

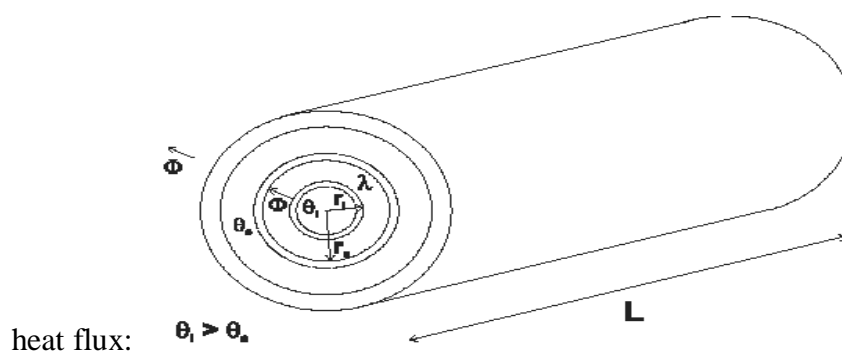


Figure 3.15.. Heat flow in concentric cylindrical tubes

$$\Phi = \frac{(\theta_i - \theta_0) 2\pi L}{\frac{1}{h_i r_i} + \frac{1}{\lambda} \ln \frac{r_e}{r_i} + \frac{1}{h_e r_e}} \quad (3-58)$$

Absolute vacuum, a cold wall can then absorb or reflect it. If it is absorbed, this radiation is transformed into heat which has the effect of increasing the temperature of the wall. 3.2 Practical coefficients. The evaluation of the thermal coefficient of convection is complex and remains the domain of the thermal engineer. In “refrigeration” practice we use coefficients h depending on the air speed and for heat exchangers we respect the manufacturers' recommendations to obtain the planned performances. Le vide absolu, une paroi froide peut alors l'absorber ou le réfléchir. S'il est absorbé, ce rayonnement se transforme en chaleur qui a pour effet d'augmenter la température de la paroi.

Average air speed [m/s]	h_0 [W/m ² .K]
0	5.6
0.5	7.6
1	9.5
2	13.5
3	17.4
4	21.4
5	25.4
6	28.5
7	32.2
8	35.7
9	39.1
10	42.4
12	48.8
14	55.0
16	61.1
18	66.8
20	72.6
22	78.2
25	86.3

4. Heat Transfer by Radiation

Radiation is fundamentally different from the other two types of heat transfer in that the substances exchanging heat do not need to be in contact with each other. They can even be separated by a vacuum. The most common manifestation of this phenomenon is solar radiation, which reaches the Earth after traveling a considerable distance through the vacuum of space.

Radiation is the emission of electromagnetic waves by a heated body, and a general explanation of the phenomenon is provided by quantum theory. In 1900, the German physicist Max Planck used quantum theory and the mathematical formalism of statistical mechanics to verify the fundamental law of radiation, known as Stefan's law. The mathematical expression of this law indicates that the total power emitted (across all wavelengths) by a heated body is proportional to T^4 , where T is the absolute temperature (expressed in K) of the body. Only a black body emits radiation that exactly satisfies Planck's law; real bodies emit with a power less than that predicted by Stefan's law.

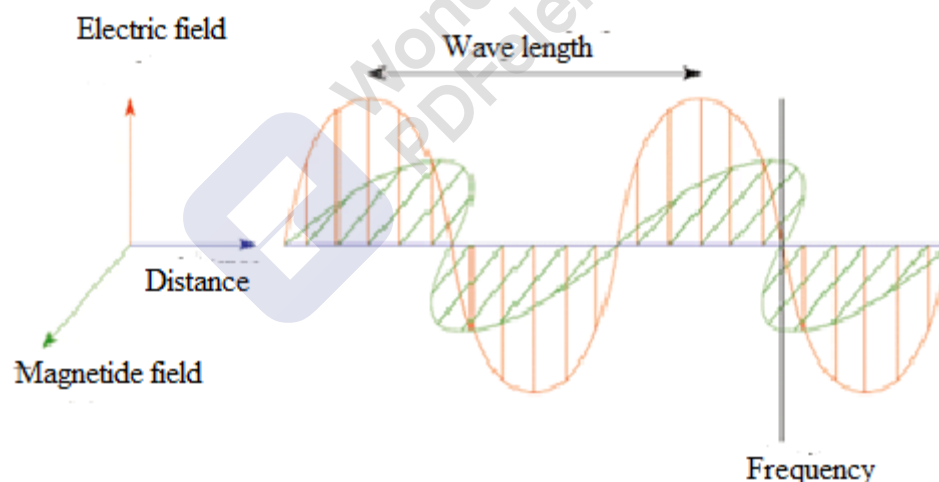


Figure 4.1. Emissivity and Black Body Radiation with Planck's Law [3]

The contribution of all frequencies to the radiant energy is called the emissive power of the body: it is the amount of energy emitted per unit area and per unit time. The proportionality factor in Stefan's law is called the Stefan-Boltzmann constant, named after the two Austrian physicists Josef Stefan and Ludwig Boltzmann, who, in 1879 and 1884 respectively, discovered the relationship between emissive power and temperature. Thus, the higher the

temperature, the greater the emitted power. In addition to emission, all substances are also capable of absorbing radiation.

Opaque surfaces can absorb or reflect incoming radiation. Generally, matte and rough surfaces absorb radiation better than shiny and polished surfaces. Conversely, shiny surfaces reflect radiation better than matte surfaces. Substances with good absorption capabilities are also powerful heat emitters, while good reflectors are poor emitters. For example, kitchen utensils have matte bottoms for good heat absorption and polished sides to minimize emission, enhancing heat transfer.

The absorption, reflection, and transmission capabilities of a substance depend on the wavelength of the incoming radiation. For instance, glass transmits large amounts of ultraviolet radiation (short waves) but poorly transmits infrared radiation (long waves).

Radiation involves a physical mechanism called electromagnetic radiation, whose propagation is almost instantaneous, at least on terrestrial scales. All solid, liquid, or gaseous bodies emit electromagnetic radiation. This energy emission occurs at the expense of their internal energy. This thermal radiation is not monochromatic. It consists of radiations with different wavelengths, ranging from 0.1 mm to 100 mm, resulting in continuous spectra for solids, or band spectra in the case of some gases. This range from 0.1 mm to 100 mm represents only a small portion of the electromagnetic spectrum, which extends from 10^{-8} mm for cosmic rays to several kilometers for radio waves.

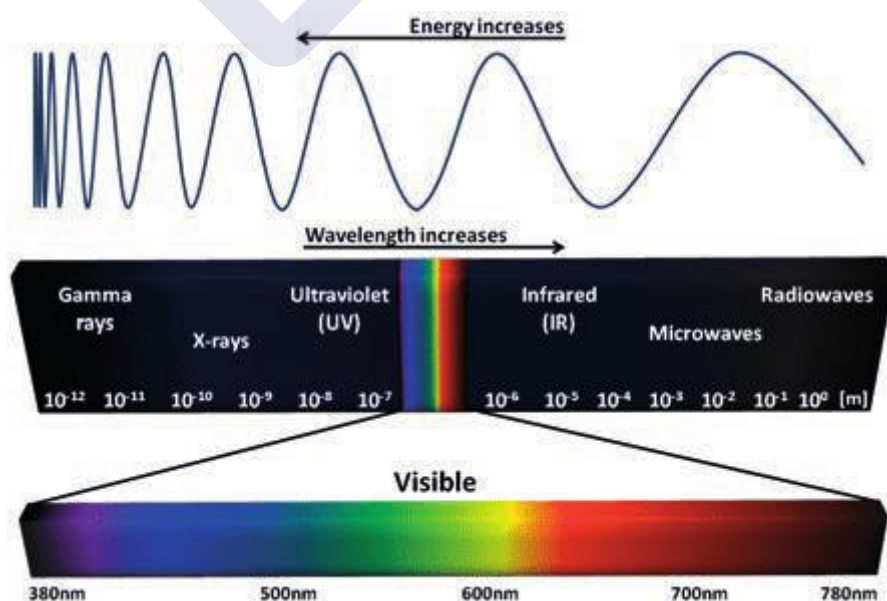


Figure 4.2. The Electromagnetic Spectrum [3]

The propagation of thermal radiation occurs in a vacuum in a straight line and at the speed of light (3×10^8 m/s), without any diminution of the transported energy. Therefore, a vacuum is considered a perfectly transparent medium.

Most simple gases (O_2 , H_2 , N_2) are also perfectly transparent mediums. However, some compound gases (particularly CO_2 , H_2O , CO) are considered partially transparent because their propagation is accompanied by a decrease in transported energy, which increases the internal energy of the gas they traverse. Certain liquids and solids (plastics, glass) also fall into this category.

The vast majority of liquids and solids, on the other hand, are considered opaque because they stop the propagation of any radiation at their surface. An incident radiation Φ that arrives on an opaque body is partially reflected Φ_r , while the rest is absorbed Φ_a as heat near the point of impact..

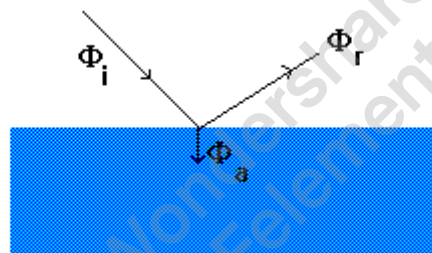


Figure 4.3. Interaction of Thermal Radiation with an Opaque Body

As in optics, reflection can be diffuse Φ_r in all directions, specular Φ_r in the direction symmetric to Φ_i , or arbitrary.

When thermal radiation interacts with an opaque body, the following interactions occur:

Diffuse Reflection: The incident radiation Φ_i is reflected in multiple directions.

Specular Reflection: The incident radiation Φ_i is reflected in a single direction, symmetric to the incident angle.

Absorption: Part of the incident radiation Φ_i is absorbed by the material, increasing its internal energy.

These interactions determine the overall behavior of the material in response to incident thermal radiation, influencing its thermal properties and heat transfer characteristics.

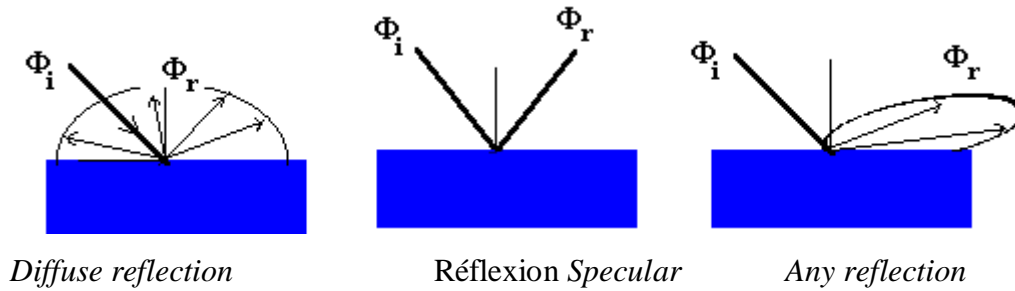


Figure 4.4. Various Types of Reflections

The Stefan-Boltzmann law provides the intensity of radiation (M in W/m^2) as a function of the absolute temperature T of the body:

$$M = \varepsilon \cdot \sigma \cdot T^4 \quad (4-1)$$

$\sigma = 5,6697 \cdot 10^{-8} \text{ W} \cdot \text{m}^{-2} \cdot \text{K}^{-4}$, c'est la constante de Stefan [5]

where:

M is the emissive power,

σ is the Stefan-Boltzmann constant $\sigma = 5,6697 \cdot 10^{-8} \text{ W} \cdot \text{m}^{-2} \cdot \text{K}^{-4}$

T is the absolute temperature of the body in Kelvin (K).

This law indicates that the total energy radiated per unit surface area of a black body is directly proportional to the fourth power of the black body's absolute temperature.

In the profession, only solar radiation is taken into account, particularly through glazing in air conditioning or in refrigerated warehouses.

The emissivity of the body, denoted ε , reveals its capacity to absorb and emit energy. A black and matte surface will have a high emissivity and a low reflection coefficient while a white and shiny surface will have the opposite behavior.

We clearly notice through this relationship that the radiation is proportional to the temperature of the body and its emissivity. So in the design of a radiator or an exchanger, the surface in contact with the fluid must be as matte and black as possible so that the radiation is better. In this specific case, thanks to radiation we can obtain additional thermal transfer. In the profession, only solar radiation is taken into account, in particular through glazing in air conditioning or in cold storage warehouses.

4.1

Material	Temperature	Emissivity
Aluminum, bare after rolling	170°C	0.04
Aluminum, black anodized	50°C	0.98
Concrete	20°C	0.93
Glaze, lisse	0°C	0.97
Iron, polished	20°C	0.24
Glass	90°C	0.94
Copper, slightly tarnished	20°C	0.04
Copper, oxidized	130°C	0.76
Copper, slightly tarnished	200°C	0.52
Hardened steel	200°C	0.79
Matte black varnish	130°C	0.97
Matte black varnish	130°C	0.97

Stefan-Boltzmann law

The total power radiated per unit surface of the black body, called total exitance, is calculated by integrating Planck's formula over the hemisphere and over all wavelengths [4]:

$$M^0(T) = \iint_{\Omega} \int_{\lambda} M^0(T, \lambda) d\Omega d\lambda = \sigma \cdot T^4 \text{ [W m}^{-2}\text{]} \quad (4-2)$$

with $\sigma = 5,6697 \cdot 10^{-8} \text{ W.m}^{-2} \cdot \text{K}^{-4}$ Stefan-Boltzmann constant

4.2 Gray Body

A black body is an ideal body. In nature and at the same temperature, most surfaces emit less than a black body: $M(T) = \varepsilon \sigma T^4$ with $0 < \varepsilon < 1$

ε = emissivity of the body

$$\varepsilon = M(T)_{\text{gray}} / M^0(T)_{\text{black}}$$

we speak of a gray body when ε is independent of the wavelength $\varepsilon_{\lambda} = \varepsilon / \varepsilon_{Ox, \lambda} = \varepsilon_{Ox}$

we speak of a diffusing body if $\varepsilon_{Ox} = \varepsilon / \varepsilon_{Ox, \lambda} = \varepsilon_{\lambda}$

gray body and diffusing $\varepsilon_{Ox, \lambda} = \varepsilon$

emitted radiation flux density $[\text{W.m}^{-2}]$

$$\varepsilon = \frac{M(T)}{M^0(T)} \Rightarrow M(T) = \varepsilon M^0(T) \rightarrow M(T) = \varepsilon \cdot \sigma T^4 \quad (4-3)$$

4.3 Interaction with matter

→ Φ_i = total incident flow

→ Φ_a = total absorbed flow

→ Φ_r = total reflected flow

→ Φ_t = total transmitted flow

→ absorptance $\alpha = \Phi_a / \Phi_i$

→ reflectance $= \Phi_r / \Phi_i$

→ transmittance $= \Phi_t / \Phi_i$

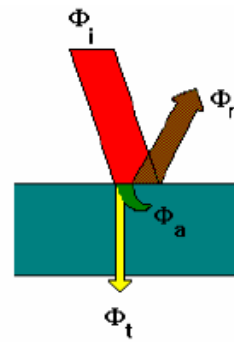


Figure 4.5. Energy conservation

$$\rightarrow \Phi_i = \Phi_a + \Phi_r + \Phi_t \rightarrow 1 = \alpha + \rho + \tau$$

If the optical properties of a material depend on the wavelength, we can cut the wavelength domain to obtain constant properties over a domain (case of the window opposite)

IR [3-10 μm] : $\alpha=0,65$ $\rho=0,30$ $\tau=0,05$ (\rightarrow serres)[4]

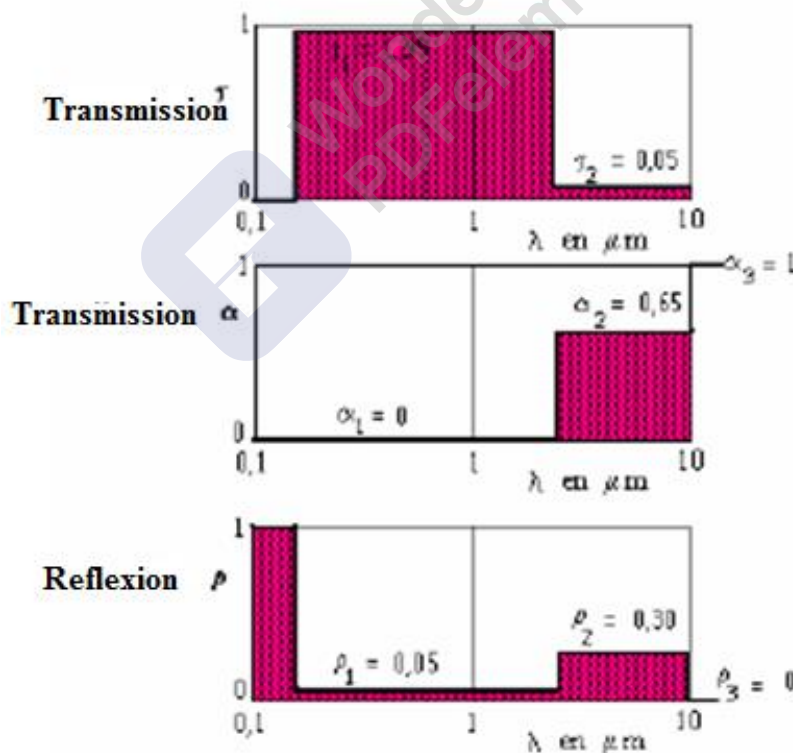


Figure 4.6. Wavelength domain [4]

4.3.1 Kirchhoff's laws:

$\lambda_\alpha = \lambda_\varepsilon$ and $\lambda_{\alpha, \alpha} = \lambda_{\alpha, \varepsilon}$ The $\alpha = \varepsilon \rightarrow \alpha = \lambda_\alpha$ and $\varepsilon = \lambda_\varepsilon = 1$ $\alpha = \varepsilon = \lambda_\varepsilon$:

$\forall \text{body } \varepsilon_{Ox,\lambda} = \alpha_{Ox,\lambda} \text{ et } \varepsilon_{\lambda} = \alpha_{\lambda}$

Gray Body: $\varepsilon_{\lambda} = \varepsilon$ et $\alpha_{\lambda} = \alpha \rightarrow \varepsilon = \alpha$

Black body: $\varepsilon_{\lambda} = \varepsilon = \alpha = 1$

4.3.2 Radiative exchanges

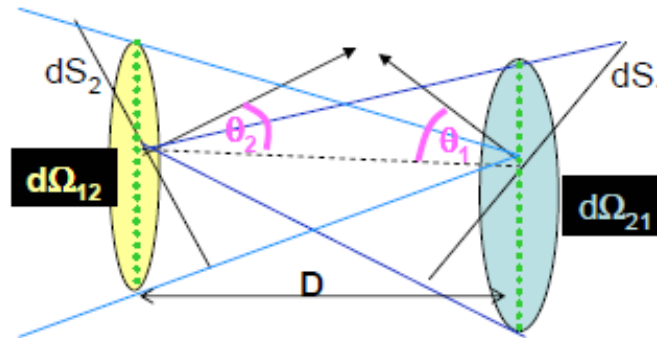


Figure 4.7. Radiative exchanges between two surfaces assimilated to black bodies

Total flow emitted by S1 $\rightarrow \Phi_1 = M_1^0 S_1$

Only a fraction reaches S2 $\rightarrow \Phi_{12} = F_{12} \Phi_1$

Total flow emitted by S2 $\rightarrow \Phi_2 = M_2^0 S_2$

Only a fraction reaches S1 $\rightarrow \Phi_{21} = F_{21} \Phi_2$

F_{ij} = geometric form factor

Reciprocity theorem: $S_i F_{ij} = S_j F_{ji}$

$$\begin{aligned} \Phi_{12} &= M_1^0 S_1 F_{12} = M_1^0 S_2 F_{21} \\ \Phi_{21} &= M_2^0 S_2 F_{21} = M_2^0 S_1 F_{12} \end{aligned} \quad (4-4)$$

$$\Phi_{\text{échange}} = \Phi_{1\text{net}} = \Phi_{2\text{net}} = S_i F_{ij} (T_i^4 - T_j^4) = S_j F_{ji} (T_i^4 - T_j^4) \quad (4-5)$$

$\rightarrow T_i > T_j \rightarrow \Phi_{\text{échange}} > 0 \rightarrow S_1$ loses energy

reciprocity $S_i F_{ij} = S_j F_{ji}$

additivity $\sum_{j=1}^n F_{ij} = 1$ F_{ii} : exchange of S_i with itself \rightarrow concave surfaces

4.3.2.1 Obvious form factors

The flux emitted by one is totally absorbed by the other $\rightarrow F_{12} = F_{21} = 1$

The flow emitted by S1 is totally absorbed by S2 $\rightarrow F_{12} = 1$

S2 concave surface $\rightarrow S_1 F_{12} = S_2 F_{21} \rightarrow F_{21} = S_1 / S_2 \rightarrow F_{22} = 1 - F_{21} = 1 - S_1 / S_2$

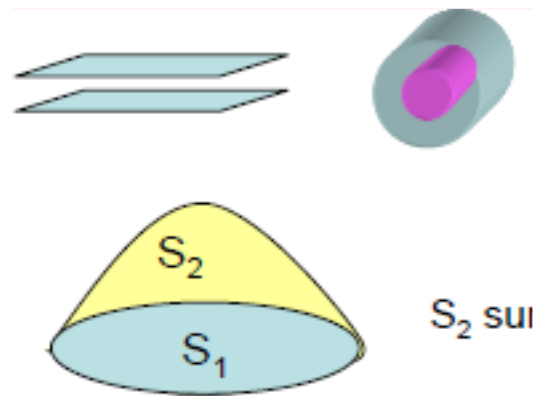


Figure 4.8. Evidence of the form factor

4.3.3 Radiation exchange between two gray surfaces

Midfield 1: S_1, ϵ_1

Midfield 2: S_2, ϵ_2

much more complex \rightarrow concept of radiosity

\rightarrow we limit ourselves to presenting a few useful cases

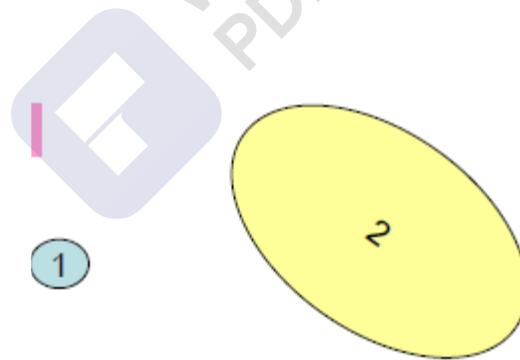


Figure 4.9 Two exchange surfaces.

$$\Phi_{12net} = S_1 \overline{F}_{12} (M_1^0 - M_2^0) = \sigma S_1 \overline{F}_{12} (T_1^4 - T_2^4)$$

$$\overline{F}_{12} = \frac{1}{\frac{1-\epsilon_1}{\epsilon_1} + \frac{1}{F_{12}} + \frac{1-\epsilon_2}{\epsilon_2} \cdot \frac{S_1}{S_2}} \quad (4-6)$$

Facteur de forme gris entre S_1 et S_2

- dépend de la géométrie

-dépend des propriétés radiatives des surfaces

Cas d'une surface convexe S_1 totalement entourée d'une surface concave S_2

S_1 ne peut rayonner sur elle-même $\rightarrow F_{11}=0$ donc $F_{12}=1$ (additivité)

Gray form factor between S_1 and S_2

- depends on geometry

-depends on the radiative properties of the surfaces

Case of a convex surface S_1 totally surrounded by a concave surface S_2

S_1 cannot radiate on itself $\rightarrow F_{11}=0$ therefore $F_{12}=1$ (additivity)

$$\overline{F_{12}} = \left[\frac{1}{\varepsilon_1} + \left(\frac{1}{\varepsilon_2 - 1} \right) \frac{S_1}{S_2} \right]^{-1} \quad (4-7)$$

Case of a convex surface S_1 totally surrounded by a concave surface S_2 , $S_2 \gg S_1$

$$\overline{F_{12}} = \frac{1}{\frac{1}{\varepsilon_1}} = \varepsilon_1 \quad \Phi_{1net} = -\Phi_{2net} = \sigma \varepsilon_1 S_1 (T_1^4 - T_2^4) \quad (4-8)$$

Case of a convex surface S_1 totally surrounded by a concave surface S_2 , $S_2 \gg S_1$

$$\overline{F_{12}} = \varepsilon_1 \quad \Phi_{1net} = -\Phi_{2net} = \sigma \varepsilon_1 S_1 (T_1^4 - T_2^4) \quad (4-9)$$

Case of 2 surfaces // at a short distance / to their dimensions

$$F_{12} = 1 \text{ et } S_1 = S_2 \text{ et } \overline{F_{12}} = \left[\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1 \right]^{-1}$$

4.4 Overall heat exchange coefficient K

- Construction of the wall: Masonry, insulation on the interior side (building type).

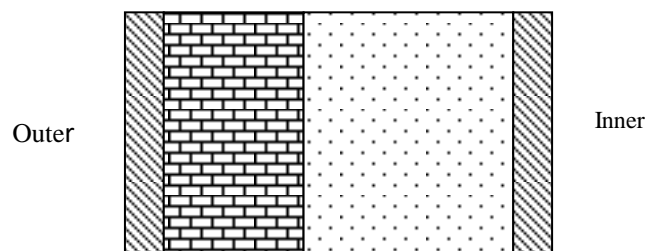


Figure 4.5 Constitution of the wall





mark	material	thickness
	Ventilated exterior cement coating 4m/s	2 cm
	Hollow body chipboard	12.5 cm
	Polystyrene insulation	10 cm
		
	Ventilated interior cement coating 1m/s	2 cm

Figure 4.6 Dimension of the Constitution of the wall

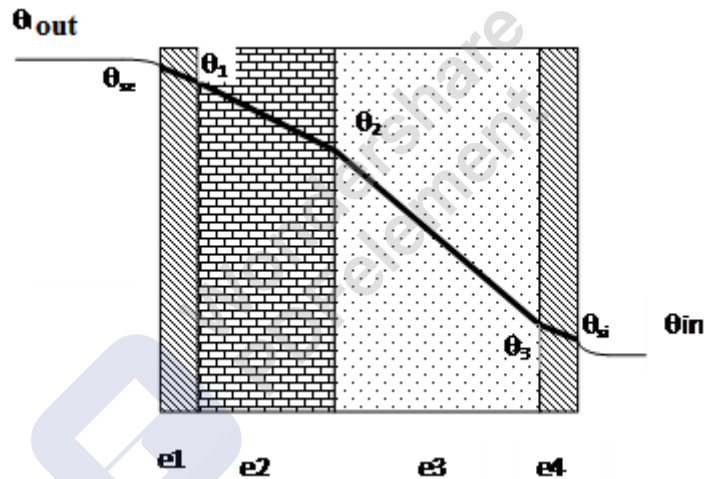


Figure 4.7. Evolution of the temperature through the wall [5]

The curve defined point by point, upon crossing each layer according to the law of heat transfer (Fourier). The density of the heat flux passing through the wall:

$$\begin{aligned}
 \varphi_u &= h_e \cdot (\theta_e - \theta_{se}) \\
 &= \lambda_1 / e_1 \cdot (\theta_{se} - \theta_1) \\
 &= \lambda_2 / e_2 \cdot (\theta_1 - \theta_2) \\
 &= \lambda_3 / e_3 \cdot (\theta_2 - \theta_3) \\
 &= \lambda_4 / e_4 \cdot (\theta_3 - \theta_{si}) \\
 &= h_i \cdot (\theta_{si} - \theta_i)
 \end{aligned}
 \tag{4-10}$$

In the case of a wall made up of several homogeneous “walls” joined together, the thermal resistance of the entire wall is equal to the sum of the thermal resistances of the various constituent walls. $R_{th} = \sum r_{th}$.

Thus, taking into account convection and radiation, we obtain: *ermiques des diverses parois constituantes.*

Thus, taking into account convection and radiation, we obtain:

$$\phi = \frac{(\theta_1 - \theta_2)}{\frac{1}{h_e} + \frac{e_1}{\lambda_1} + \frac{e_2}{\lambda_2} + \frac{e_3}{\lambda_3} + \frac{e_4}{\lambda_4} + \frac{1}{h_i}} \quad (4-11)$$

$$\text{and } K = \left(\frac{1}{\frac{1}{h_e} + \frac{e_1}{\lambda_1} + \frac{e_2}{\lambda_2} + \frac{e_3}{\lambda_3} + \frac{e_4}{\lambda_4} + \frac{1}{h_i}} \right) \quad (4-12)$$

we call K overall heat exchange coefficient.

Thermal power transmitted by a wall of surface S:

$$P_{th} = K \cdot S \cdot \Delta\theta \quad (4-13)$$

With P in Watts

K to $\text{W} \cdot \text{m}^{-2} \cdot ^\circ\text{C}^{-1}$

Temperature difference between the fluids separated by the wall. $\theta\Delta$

4.5. Exercise

A Langø of steel is loaded into an electric furnace whose wall temperature $T_p=100^\circ\text{C}$? the surface of the oven $F_2 \gg F_1$ the two bodies have the same emissivity $\varepsilon_1=\varepsilon_2=0.8$.

Calculate the radioactive flux density as a function of Langø temperature $\Phi=f(T)$

$T=20^\circ\text{C}$ 100°C 300°C 500°C 700°C ?

a) Do the same calculation assuming that F_1/F_2

Answer

The density of the net radiation flux exchanged between the two horns equal to:

$$\Phi_{12net} = \frac{C_0 \left(\left(\frac{T_2}{100} \right)^4 - \left(\frac{T_1}{100} \right)^4 \right)}{\frac{1}{\varepsilon_1} + \left(\frac{1}{\varepsilon_2} - 1 \right) \cdot \frac{F_1}{F_2}}$$

In our case we have $F_1/F_2 \ll 0$ hence $\varepsilon_{\Pi}=\varepsilon_I$ d'où



$$\Phi_{12net} = \varepsilon_1 C_0 \left(\left(\frac{T_2}{100} \right)^4 - \left(\frac{T_1}{100} \right)^4 \right)$$

The final expression for the flux density is:

$$\Phi_{12net} = 0,85,87 \left(\left(\frac{1273}{100} \right)^4 - \left(\frac{T_1}{100} \right)^4 \right) = 4,536(26860 - \left(\frac{T_1}{100} \right)^4)$$

T_1 °C	20	100	300	500	700
Φ_{12net} W/m ²	118,8	113,6	111,2	102,9	98,5

2) same step for the case of

$$F_2/F_1 = 1/5$$

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